

**$k = 4$  search results discussed in  
*D*-optimal and near *D*-optimal  $2^k$  fractional  
factorial designs of resolution V  
by Bulutoglu and Ryan (2006)**

The table below lists search results for  $k = 4$  factor designs. Column definitions are the number of runs  $N$ , the number of factors  $k$ , determinant of the information matrix  $\det(\mathbf{X}_M^T \mathbf{X}_M)$ , a lower bound for *D*-efficiency  $\underline{e}(\mathbf{X})$ , an indicator for type 1 optimality  $I_1$ , and an indicator for type 2 optimality  $I_2$ .

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$\underline{e}(\mathbf{X})$	$I_1$	$I_2$
11	4	38654705664	90.19646%	0	0
12	4	137438953472	85.82266%	0	0
13	4	481036337152	90.61858%	0	0
14	4	1.649267e+012	95.54409%	0	0
15	4	5.497558e+012	99.81041%	0	1
16	4	1.759219e+013	100.00000%	1	1
17	4	2.968681e+013	100.00000%	1	0
18	4	5.002778e+013	100.00000%	1	0
19	4	8.418136e+013	99.18046%	0	0
20	4	1.414247e+014	96.69009%	0	0
21	4	2.371810e+014	97.38520%	0	0
22	4	3.896394e+014	97.91057%	0	0
23	4	6.456882e+014	97.80492%	0	0
24	4	1.068725e+015	96.83878%	0	0
25	4	1.692148e+015	97.56549%	0	0
26	4	2.680060e+015	98.30769%	0	0
27	4	4.294967e+015	98.56461%	0	0
28	4	6.597070e+015	97.94106%	0	0

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$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
29	4	1.011551e+016	98.80034%	0	0
30	4	1.548112e+016	99.65597%	0	0
31	4	2.364390e+016	100.00000%	0	1
32	4	3.602880e+016	100.00000%	1	1
33	4	4.841370e+016	100.00000%	1	0
34	4	6.502072e+016	100.00000%	1	0
35	4	8.727483e+016	99.58064%	0	0
36	4	1.170760e+017	98.94102%	0	0
37	4	1.569550e+017	99.17973%	0	0
38	4	2.086323e+017	99.34612%	0	0
39	4	2.783139e+017	99.19810%	0	0
40	4	3.710852e+017	98.89285%	0	0
41	4	4.870493e+017	99.15380%	0	0
42	4	6.391061e+017	99.41684%	0	0
43	4	8.422505e+017	99.42394%	0	0
44	4	1.095610e+018	99.20100%	0	0
45	4	1.424294e+018	99.55534%	0	0
46	4	1.850364e+018	99.90797%	0	0
47	4	2.402227e+018	100.00000%	0	1
48	4	3.116403e+018	100.00000%	1	1
49	4	3.830579e+018	100.00000%	1	0
50	4	4.707067e+018	100.00000%	1	0
51	4	5.782388e+018	99.77606%	0	0
52	4	7.101179e+018	99.48417%	0	0
53	4	8.717965e+018	99.60393%	0	0

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$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
54	4	1.065352e+019	99.68424%	0	0
55	4	1.304388e+019	99.58877%	0	0
56	4	1.596628e+019	99.43923%	0	0
57	4	1.938763e+019	99.57325%	0	0
58	4	2.353787e+019	99.70783%	0	0
59	4	2.863967e+019	99.69192%	0	0
60	4	3.463043e+019	99.57793%	0	0
61	4	4.186096e+019	99.77081%	0	0
62	4	5.058443e+019	99.96285%	0	0
63	4	6.110484e+019	100.00000%	0	1
64	4	7.378698e+019	100.00000%	1	1
65	4	8.646911e+019	100.00000%	1	0
66	4	1.013130e+020	100.00000%	1	0
67	4	1.186834e+020	99.86248%	0	0
68	4	1.390059e+020	99.69558%	0	0
69	4	1.627768e+020	99.76742%	0	0
70	4	1.900621e+020	99.81452%	0	0
71	4	2.221880e+020	99.75050%	0	0
72	4	2.597002e+020	99.66178%	0	0
73	4	3.020819e+020	99.74331%	0	0
74	4	3.513346e+020	99.82506%	0	0
75	4	4.091650e+020	99.80863%	0	0
76	4	4.747764e+020	99.73951%	0	0
77	4	5.508027e+020	99.86070%	0	0
78	4	6.388764e+020	99.98144%	0	0

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$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
79	4	7.408819e+020	100.00000%	0	1
80	4	8.589935e+020	100.00000%	1	1
81	4	9.771051e+020	100.00000%	1	0
82	4	1.111323e+021	100.00000%	1	0
83	4	1.263821e+021	99.90731%	0	0
84	4	1.437064e+021	99.79933%	0	0
85	4	1.633844e+021	99.84718%	0	0
86	4	1.853894e+021	99.87808%	0	0
87	4	2.105310e+021	99.83265%	0	0
88	4	2.390546e+021	99.77392%	0	0
89	4	2.705663e+021	99.82875%	0	0
90	4	3.062028e+021	99.88366%	0	0
91	4	3.468443e+021	99.86970%	0	0
92	4	3.919396e+021	99.82334%	0	0
93	4	4.428409e+021	99.90652%	0	0
94	4	5.002866e+021	99.98943%	0	0
95	4	5.651077e+021	100.00000%	0	1
96	4	6.382393e+021	100.00000%	1	1
97	4	7.113709e+021	100.00000%	1	0
98	4	7.928129e+021	100.00000%	1	0
99	4	8.835002e+021	99.93339%	0	0
100	4	9.844717e+021	99.85783%	0	0
101	4	1.096881e+022	99.89198%	0	0
102	4	1.220368e+022	99.91379%	0	0
103	4	1.358566e+022	99.88001%	0	0

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$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
104	4	1.512288e+022	99.83827%	0	0
105	4	1.679512e+022	99.87766%	0	0
106	4	1.865093e+022	99.91709%	0	0
107	4	2.072514e+022	99.90560%	0	0
108	4	2.299109e+022	99.87235%	0	0
109	4	2.550242e+022	99.93297%	0	0
110	4	2.828540e+022	99.99341%	0	0
111	4	3.136907e+022	100.00000%	0	1
112	4	3.478550e+022	100.00000%	1	1
113	4	3.820193e+022	100.00000%	1	0
114	4	4.195114e+022	100.00000%	1	0
115	4	4.606520e+022	99.94985%	0	0
116	4	5.057926e+022	99.89402%	0	0
117	4	5.553181e+022	99.91961%	0	0
118	4	6.090263e+022	99.93583%	0	0
119	4	6.682316e+022	99.90978%	0	0
120	4	7.331456e+022	99.87859%	0	0
121	4	8.029689e+022	99.90825%	0	0
122	4	8.793928e+022	99.93794%	0	0
123	4	9.635564e+022	99.92847%	0	0
124	4	1.054448e+023	99.90347%	0	0
125	4	1.153832e+023	99.94961%	0	0
126	4	1.262495e+023	99.99562%	0	0
127	4	1.381292e+023	100.00000%	0	1
128	4	1.511157e+023	100.00000%	1	1

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$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
129	4	1.641022e+023	100.00000%	1	0
130	4	1.781955e+023	100.00000%	1	0
131	4	1.934891e+023	99.96090%	0	0
132	4	2.100839e+023	99.91797%	0	0
133	4	2.280897e+023	99.93786%	0	0
134	4	2.474261e+023	99.95039%	0	0
135	4	2.684971e+023	99.92970%	0	0
136	4	2.913479e+023	99.90551%	0	0
137	4	3.157162e+023	99.92865%	0	0
138	4	3.421072e+023	99.95181%	0	0
139	4	3.708440e+023	99.94394%	0	0
140	4	4.016041e+023	99.92446%	0	0
141	4	4.348921e+023	99.96074%	0	0
142	4	4.709135e+023	99.99694%	0	0
143	4	5.098901e+023	100.00000%	0	1
144	4	5.520614e+023	100.00000%	1	1
145	4	5.942328e+023	100.00000%	1	0
146	4	6.395990e+023	100.00000%	1	0
147	4	6.883995e+023	99.96867%	0	0
148	4	7.408919e+023	99.93463%	0	0
149	4	7.973524e+023	99.95053%	0	0
150	4	8.575213e+023	99.96050%	0	0
151	4	9.224948e+023	99.94369%	0	0
152	4	9.923511e+023	99.92438%	0	0
153	4	1.066342e+024	99.94294%	0	0

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$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
154	4	1.145808e+024	99.96151%	0	0
155	4	1.231567e+024	99.95488%	0	0
156	4	1.322722e+024	99.93927%	0	0
157	4	1.420562e+024	99.96856%	0	0
158	4	1.525572e+024	99.99778%	0	0
159	4	1.638272e+024	100.00000%	0	1
160	4	1.759219e+024	100.00000%	1	1
161	4	1.880165e+024	100.00000%	1	0
162	4	2.009357e+024	100.00000%	1	0
163	4	2.147353e+024	99.97433%	0	0
164	4	2.294745e+024	99.94668%	0	0
165	4	2.452168e+024	99.95968%	0	0
166	4	2.618895e+024	99.96780%	0	0
167	4	2.797619e+024	99.95388%	0	0
168	4	2.988439e+024	99.93811%	0	0
169	4	3.189447e+024	99.95332%	0	0
170	4	3.403871e+024	99.96854%	0	0
171	4	3.633608e+024	99.96291%	0	0
172	4	3.876408e+024	99.95012%	0	0
173	4	4.135286e+024	99.97426%	0	0
174	4	4.411293e+024	99.99834%	0	0
175	4	4.705552e+024	100.00000%	0	1
176	4	5.019256e+024	100.00000%	1	1
177	4	5.332959e+024	100.00000%	1	0
178	4	5.666107e+024	100.00000%	1	0

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$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
179	4	6.019893e+024	99.97859%	0	0
180	4	6.395582e+024	99.95568%	0	0
181	4	6.794515e+024	99.96652%	0	0
182	4	7.214886e+024	99.97325%	0	0
183	4	7.662778e+024	99.96153%	0	0
184	4	8.138246e+024	99.94841%	0	0
185	4	8.636834e+024	99.96111%	0	0
186	4	9.165731e+024	99.97381%	0	0
187	4	9.729021e+024	99.96897%	0	0
188	4	1.032153e+025	99.95830%	0	0
189	4	1.094979e+025	99.97853%	0	0
190	4	1.161596e+025	99.99873%	0	0
191	4	1.232227e+025	100.00000%	0	1
192	4	1.307114e+025	100.00000%	1	1
193	4	1.382001e+025	100.00000%	1	0
194	4	1.461143e+025	100.00000%	1	0
195	4	1.544779e+025	99.98187%	0	0
196	4	1.633161e+025	99.96259%	0	0
197	4	1.726558e+025	99.97175%	0	0
198	4	1.824555e+025	99.97743%	0	0
199	4	1.928437e+025	99.96743%	0	0
200	4	2.038185e+025	99.95634%	0	0
201	4	2.152833e+025	99.96710%	0	0
202	4	2.273880e+025	99.97786%	0	0
203	4	2.402151e+025	99.97366%	0	0

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$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
204	4	2.536539e+025	99.96462%	0	0
205	4	2.678378e+025	99.98183%	0	0
206	4	2.828077e+025	99.99900%	0	0
207	4	2.986067e+025	100.00000%	0	1
208	4	3.152802e+025	100.00000%	1	1
209	4	3.319537e+025	100.00000%	1	0
210	4	3.495016e+025	100.00000%	1	0
211	4	3.679695e+025	99.98445%	0	0
212	4	3.874049e+025	99.96799%	0	0
213	4	4.078582e+025	99.97584%	0	0
214	4	4.292414e+025	99.98070%	0	0
215	4	4.518108e+025	99.97206%	0	0
216	4	4.755573e+025	99.96257%	0	0
217	4	5.002835e+025	99.97180%	0	0
218	4	5.262853e+025	99.98104%	0	0
219	4	5.537214e+025	99.97736%	0	0
220	4	5.823679e+025	99.96961%	0	0
221	4	6.124834e+025	99.98442%	0	0
222	4	6.441423e+025	99.99920%	0	0
223	4	6.774228e+025	100.00000%	0	1
224	4	7.124070e+025	100.00000%	1	1
225	4	7.473913e+025	100.00000%	1	0
226	4	7.840794e+025	100.00000%	1	0
227	4	8.225533e+025	99.98652%	0	0
228	4	8.628993e+025	99.97231%	0	0

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$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
229	4	9.052074e+025	99.97911%	0	0
230	4	9.493017e+025	99.98331%	0	0
231	4	9.956687e+025	99.97578%	0	0
232	4	1.044282e+026	99.96756%	0	0
233	4	1.094760e+026	99.97557%	0	0
234	4	1.147658e+026	99.98358%	0	0
235	4	1.203269e+026	99.98033%	0	0
236	4	1.261163e+026	99.97361%	0	0
237	4	1.321818e+026	99.98649%	0	0
238	4	1.385364e+026	99.99935%	0	0
239	4	1.451937e+026	100.00000%	0	1
240	4	1.521681e+026	100.00000%	1	1
241	4	1.591425e+026	100.00000%	1	0
242	4	1.664339e+026	100.00000%	1	0
243	4	1.740565e+026	99.98820%	0	0
244	4	1.820254e+026	99.97580%	0	0
245	4	1.903560e+026	99.98175%	0	0
246	4	1.990148e+026	99.98542%	0	0
247	4	2.080904e+026	99.97880%	0	0
248	4	2.175765e+026	99.97161%	0	0
249	4	2.274026e+026	99.97863%	0	0
250	4	2.376688e+026	99.98564%	0	0