

$k = 7$ search results discussed in
D-optimal and near *D*-optimal 2^k fractional
 factorial designs of resolution V
 by Bulutoglu and Ryan (2006)

The table below lists search results for $k = 7$ factor designs. Column definitions are the number of runs N , the number of factors k , determinant of the information matrix $\det(\mathbf{X}_M^T \mathbf{X}_M)$, a lower bound for *D*-efficiency $e(\mathbf{X})$, an indicator for type 1 optimality I_1 , and an indicator for type 2 optimality I_2 . For $N \equiv 1$ or $2 \pmod{16}$, type 1 optimality is restricted generalized type 1 optimality as studied in Chadjiconstantinidis et al. (1989). For the remaining cases, generalized type 1 optimality is as in Cheng (1980).

| N | k | $\det(\mathbf{X}_M^T \mathbf{X}_M)$ | $e(\mathbf{X})$ | I_1 | I_2 |
|-----|-----|-------------------------------------|-----------------|-------|-------|
| 29 | 7 | 2.85221385051351e+40 | 86.5372% | 0 | 0 |
| 30 | 7 | 9.49074158758389e+40 | 88.01691% | 0 | 0 |
| 31 | 7 | 3.9571831894565e+41 | 90.00273% | 0 | 0 |
| 32 | 7 | 1.36865289813990e+42 | 88.67999% | 0 | 0 |
| 33 | 7 | 5.51257434411918e+42 | 90.99729% | 0 | 0 |
| 34 | 7 | 2.48065845485363e+43 | 93.73368% | 0 | 0 |
| 35 | 7 | 5.61714098870919e+43 | 94.04253% | 0 | 0 |
| 36 | 7 | 2.04888096511500e+44 | 93.68732% | 0 | 0 |
| 37 | 7 | 4.28592446784262e+44 | 94.16399% | 0 | 0 |
| 38 | 7 | 9.2759884319074e+44 | 94.75505% | 0 | 0 |
| 39 | 7 | 2.48374549648635e+45 | 95.78027% | 0 | 0 |
| 40 | 7 | 7.22544144432394e+45 | 95.3411% | 0 | 0 |
| 41 | 7 | 1.35226143697589e+46 | 95.60951% | 0 | 0 |
| 42 | 7 | 2.64857881947463e+46 | 96.02989% | 0 | 0 |
| 43 | 7 | 6.26395241235485e+46 | 96.79573% | 0 | 0 |
| 44 | 7 | 1.25279048247097e+47 | 95.63395% | 0 | 0 |

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| N | k | $\det(\mathbf{X}_M^T \mathbf{X}_M)$ | $\underline{e}(\mathbf{X})$ | I_1 | I_2 |
|-----|-----|-------------------------------------|-----------------------------|-------|-------|
| 45 | 7 | 2.50558096494194e+47 | 96.25221% | 0 | 0 |
| 46 | 7 | 5.01116192988387e+47 | 96.87509% | 0 | 0 |
| 47 | 7 | 1.00223238597677e+48 | 97.20674% | 0 | 0 |
| 48 | 7 | 2.00446477195354e+48 | 96.45948% | 0 | 0 |
| 49 | 7 | 4.00892954390709e+48 | 97.19584% | 0 | 0 |
| 50 | 7 | 8.0178590878142e+48 | 97.9384% | 0 | 0 |
| 51 | 7 | 1.60357181756283e+49 | 98.38412% | 0 | 0 |
| 52 | 7 | 2.54994527963333e+49 | 97.20087% | 0 | 0 |
| 53 | 7 | 4.51199479082791e+49 | 97.62903% | 0 | 0 |
| 54 | 7 | 8.00480761165843e+49 | 98.0685% | 0 | 0 |
| 55 | 7 | 1.39720858970699e+50 | 98.14855% | 0 | 0 |
| 56 | 7 | 2.44409665837957e+50 | 97.57382% | 0 | 0 |
| 57 | 7 | 4.35657890279278e+50 | 98.11605% | 0 | 0 |
| 58 | 7 | 7.84010573385841e+50 | 98.69428% | 0 | 0 |
| 59 | 7 | 1.31016878041367e+51 | 98.71985% | 0 | 0 |
| 60 | 7 | 2.24791511601188e+51 | 98.31047% | 0 | 0 |
| 61 | 7 | 3.93920363186844e+51 | 98.87716% | 0 | 0 |
| 62 | 7 | 7.15551201637209e+51 | 99.57083% | 0 | 0 |
| 63 | 7 | 1.30950546704849e+52 | 100% | 0 | 1 |
| 64 | 7 | 2.39452428260294e+52 | 100% | 1 | 1 |
| 65 | 7 | 3.47954309815742e+52 | 100% | 1 | 0 |
| 66 | 7 | 5.05562646385506e+52 | 100% | 1 | 0 |
| 67 | 7 | 7.337907420711e+52 | 99.71661% | 0 | 0 |
| 68 | 7 | 1.06490855302479e+53 | 99.08755% | 0 | 0 |
| 69 | 7 | 1.57453872688221e+53 | 99.21019% | 0 | 0 |

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Table 1 – continued from previous page

| N | k | $\det(\mathbf{X}_M^T \mathbf{X}_M)$ | $\underline{e}(\mathbf{X})$ | I_1 | I_2 |
|-----|-----|-------------------------------------|-----------------------------|-------|-------|
| 70 | 7 | 2.32301927132167e+53 | 99.32592% | 0 | 0 |
| 71 | 7 | 3.41144071562287e+53 | 99.16315% | 0 | 0 |
| 72 | 7 | 5.01489933542222e+53 | 98.71899% | 0 | 0 |
| 73 | 7 | 7.38816626218766e+53 | 98.88597% | 0 | 0 |
| 74 | 7 | 1.15649174349897e+54 | 99.26088% | 0 | 0 |
| 75 | 7 | 1.72040094074227e+54 | 99.21507% | 0 | 0 |
| 76 | 7 | 2.55524402763308e+54 | 98.92471% | 0 | 0 |
| 77 | 7 | 3.78967489376334e+54 | 99.16687% | 0 | 0 |
| 78 | 7 | 5.61285882374614e+54 | 99.40531% | 0 | 0 |
| 79 | 7 | 8.34957082599327e+54 | 99.42659% | 0 | 0 |
| 80 | 7 | 1.24328001743884e+55 | 99.24822% | 0 | 0 |
| 81 | 7 | 1.84823457513276e+55 | 99.54695% | 0 | 0 |
| 82 | 7 | 2.66930820970909e+55 | 99.74745% | 0 | 0 |
| 83 | 7 | 3.77137078971373e+55 | 99.65376% | 0 | 0 |
| 84 | 7 | 5.32009432186478e+55 | 99.38118% | 0 | 0 |
| 85 | 7 | 7.34703368499269e+55 | 99.47112% | 0 | 0 |
| 86 | 7 | 1.02681798052605e+56 | 99.60244% | 0 | 0 |
| 87 | 7 | 1.40089044104393e+56 | 99.44604% | 0 | 0 |
| 88 | 7 | 1.90992999309195e+56 | 99.13844% | 0 | 0 |
| 89 | 7 | 2.64239899527385e+56 | 99.27453% | 0 | 0 |
| 90 | 7 | 3.78093601202942e+56 | 99.52653% | 0 | 0 |
| 91 | 7 | 5.23041304478368e+56 | 99.47051% | 0 | 0 |
| 92 | 7 | 7.35641778450587e+56 | 99.34173% | 0 | 0 |
| 93 | 7 | 1.03690460200654e+57 | 99.57923% | 0 | 0 |
| 94 | 7 | 1.49065156478250e+57 | 99.88544% | 0 | 0 |

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| N | k | $\det(\mathbf{X}_M^T \mathbf{X}_M)$ | $\underline{e}(\mathbf{X})$ | I_1 | I_2 |
|-----|-----|-------------------------------------|-----------------------------|-------|-------|
| 95 | 7 | 2.13633486289684e+57 | 100% | 0 | 1 |
| 96 | 7 | 3.06101711698651e+57 | 100% | 1 | 1 |
| 97 | 7 | 3.98569937107619e+57 | 100% | 1 | 0 |
| 98 | 7 | 5.18938058114121e+57 | 100% | 1 | 0 |
| 99 | 7 | 6.75266044557441e+57 | 99.82719% | 0 | 0 |
| 100 | 7 | 8.79007592416706e+57 | 99.55629% | 0 | 0 |
| 101 | 7 | 1.15552197342602e+58 | 99.6211% | 0 | 0 |
| 102 | 7 | 1.53309384632180e+58 | 99.71786% | 0 | 0 |
| 103 | 7 | 1.99809007904916e+58 | 99.59288% | 0 | 0 |
| 104 | 7 | 2.60274003955899e+58 | 99.37837% | 0 | 0 |
| 105 | 7 | 3.43895400907335e+58 | 99.4903% | 0 | 0 |
| 106 | 7 | 4.63603258920014e+58 | 99.67157% | 0 | 0 |
| 107 | 7 | 6.10109916500795e+58 | 99.61894% | 0 | 0 |
| 108 | 7 | 8.1337815945155e+58 | 99.53267% | 0 | 0 |
| 109 | 7 | 1.08485885753752e+59 | 99.70497% | 0 | 0 |
| 110 | 7 | 1.44392073445497e+59 | 99.87054% | 0 | 0 |
| 111 | 7 | 1.91814256309734e+59 | 99.88651% | 0 | 0 |
| 112 | 7 | 2.54363178083309e+59 | 99.82652% | 0 | 0 |
| 113 | 7 | 3.36762517462409e+59 | 100% | 1 | 0 |
| 114 | 7 | 4.23938630138849e+59 | 100% | 1 | 0 |
| 115 | 7 | 5.33438846592646e+59 | 99.86306% | 0 | 0 |
| 116 | 7 | 6.71705017188663e+59 | 99.66626% | 0 | 0 |
| 117 | 7 | 8.51672366788826e+59 | 99.71521% | 0 | 0 |
| 118 | 7 | 1.08776904654494e+60 | 99.78946% | 0 | 0 |
| 119 | 7 | 1.37038408140080e+60 | 99.68865% | 0 | 0 |

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| N | k | $\det(\mathbf{X}_M^T \mathbf{X}_M)$ | $\underline{e}(\mathbf{X})$ | I_1 | I_2 |
|-----|-----|-------------------------------------|-----------------------------|-------|-------|
| 120 | 7 | 1.72569594870125e+60 | 99.53033% | 0 | 0 |
| 121 | 7 | 2.20096944805406e+60 | 99.6224% | 0 | 0 |
| 122 | 7 | 2.84365252688585e+60 | 99.75915% | 0 | 0 |
| 123 | 7 | 3.6082354520714e+60 | 99.7127% | 0 | 0 |
| 124 | 7 | 4.62590014351601e+60 | 99.65099% | 0 | 0 |
| 125 | 7 | 5.94127679254349e+60 | 99.78887% | 0 | 0 |
| 126 | 7 | 7.6894496258487e+60 | 99.95353% | 0 | 0 |
| 127 | 7 | 9.9429291488525e+60 | 100% | 0 | 1 |
| 128 | 7 | 1.28555043540719e+61 | 100% | 1 | 1 |
| 129 | 7 | 1.57680795592913e+61 | 100% | 1 | 0 |
| 130 | 7 | 1.93397504467264e+61 | 100% | 1 | 0 |
| 131 | 7 | 2.37117518054231e+61 | 99.88935% | 0 | 0 |
| 132 | 7 | 2.90961759418462e+61 | 99.73984% | 0 | 0 |
| 133 | 7 | 3.58857517468088e+61 | 99.77814% | 0 | 0 |
| 134 | 7 | 4.4521941944628e+61 | 99.83691% | 0 | 0 |
| 135 | 7 | 5.46386649029378e+61 | 99.75438% | 0 | 0 |
| 136 | 7 | 6.70311279688632e+61 | 99.63265% | 0 | 0 |
| 137 | 7 | 8.31538492070412e+61 | 99.7091% | 0 | 0 |
| 138 | 7 | 1.04065195511599e+62 | 99.81599% | 0 | 0 |
| 139 | 7 | 1.28417293894475e+62 | 99.77567% | 0 | 0 |
| 140 | 7 | 1.59803693914038e+62 | 99.72941% | 0 | 0 |
| 141 | 7 | 1.99140068444453e+62 | 99.83885% | 0 | 0 |
| 142 | 7 | 2.49545720876696e+62 | 99.96773% | 0 | 0 |
| 143 | 7 | 3.12498271152304e+62 | 100% | 0 | 1 |
| 144 | 7 | 3.91302183008102e+62 | 100% | 1 | 1 |

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| N | k | $\det(\mathbf{X}_M^T \mathbf{X}_M)$ | $\underline{e}(\mathbf{X})$ | I_1 | I_2 |
|-----|-----|-------------------------------------|-----------------------------|-------|-------|
| 145 | 7 | 4.70106094863902e+62 | 100% | 1 | 0 |
| 146 | 7 | 5.64761368299889e+62 | 100% | 1 | 0 |
| 147 | 7 | 6.78270536356124e+62 | 99.90895% | 0 | 0 |
| 148 | 7 | 8.15272871355165e+62 | 99.7915% | 0 | 0 |
| 149 | 7 | 9.8378124356494e+62 | 99.82228% | 0 | 0 |
| 150 | 7 | 1.19291859766497e+63 | 99.86996% | 0 | 0 |
| 151 | 7 | 1.43378215666235e+63 | 99.80136% | 0 | 0 |
| 152 | 7 | 1.72278662594672e+63 | 99.70481% | 0 | 0 |
| 153 | 7 | 2.0901520895291e+63 | 99.76905% | 0 | 0 |
| 154 | 7 | 2.55172664907869e+63 | 99.85491% | 0 | 0 |
| 155 | 7 | 3.0803622416644e+63 | 99.82% | 0 | 0 |
| 156 | 7 | 3.74443336569256e+63 | 99.78405% | 0 | 0 |
| 157 | 7 | 4.55675972305016e+63 | 99.87301% | 0 | 0 |
| 158 | 7 | 5.56873838077861e+63 | 99.97667% | 0 | 0 |
| 159 | 7 | 6.80190888468064e+63 | 100% | 0 | 1 |
| 160 | 7 | 8.30767497365571e+63 | 100% | 1 | 1 |
| 161 | 7 | 9.8134410626308e+63 | 100% | 1 | 0 |
| 162 | 7 | 1.15918027366790e+64 | 100% | 1 | 0 |
| 163 | 7 | 1.36889714378412e+64 | 99.92388% | 0 | 0 |
| 164 | 7 | 1.61784422297165e+64 | 99.82916% | 0 | 0 |
| 165 | 7 | 1.91796045625066e+64 | 99.85444% | 0 | 0 |
| 166 | 7 | 2.28305960724539e+64 | 99.8939% | 0 | 0 |
| 167 | 7 | 2.69782234170139e+64 | 99.83608% | 0 | 0 |
| 168 | 7 | 3.18716778565e+64 | 99.75762% | 0 | 0 |
| 169 | 7 | 3.79710795150458e+64 | 99.81221% | 0 | 0 |

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| N | k | $\det(\mathbf{X}_M^T \mathbf{X}_M)$ | $\underline{e}(\mathbf{X})$ | I_1 | I_2 |
|-----|-----|-------------------------------------|-----------------------------|-------|-------|
| 170 | 7 | 4.54455443131456e+64 | 99.88271% | 0 | 0 |
| 171 | 7 | 5.38921990024181e+64 | 99.85237% | 0 | 0 |
| 172 | 7 | 6.42829024001531e+64 | 99.82365% | 0 | 0 |
| 173 | 7 | 7.67469376815368e+64 | 99.89738% | 0 | 0 |
| 174 | 7 | 9.19298607263399e+64 | 99.98259% | 0 | 0 |
| 175 | 7 | 1.10070777337647e+65 | 100% | 0 | 1 |
| 176 | 7 | 1.3178542048589e+65 | 100% | 1 | 1 |
| 177 | 7 | 1.53500063634134e+65 | 100% | 1 | 0 |
| 178 | 7 | 1.78788433319313e+65 | 100% | 1 | 0 |
| 179 | 7 | 2.08198191530206e+65 | 99.93547% | 0 | 0 |
| 180 | 7 | 2.42626061488221e+65 | 99.85746% | 0 | 0 |
| 181 | 7 | 2.83450701818376e+65 | 99.8786% | 0 | 0 |
| 182 | 7 | 3.32296837907265e+65 | 99.91179% | 0 | 0 |
| 183 | 7 | 3.87164807474683e+65 | 99.86246% | 0 | 0 |
| 184 | 7 | 4.50999746540756e+65 | 99.79741% | 0 | 0 |
| 185 | 7 | 5.29244587265098e+65 | 99.84432% | 0 | 0 |
| 186 | 7 | 6.2321736261701e+65 | 99.90325% | 0 | 0 |
| 187 | 7 | 7.28253996765946e+65 | 99.87674% | 0 | 0 |
| 188 | 7 | 8.5523379434513e+65 | 99.85328% | 0 | 0 |
| 189 | 7 | 1.00511445589333e+66 | 99.91536% | 0 | 0 |
| 190 | 7 | 1.1843826521662e+66 | 99.98666% | 0 | 0 |
| 191 | 7 | 1.39515354871249e+66 | 100% | 0 | 1 |
| 192 | 7 | 1.64337105124417e+66 | 100% | 1 | 1 |
| 193 | 7 | 1.89158855377583e+66 | 100% | 1 | 0 |
| 194 | 7 | 2.17725266229289e+66 | 100% | 1 | 0 |

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| N | k | $\det(\mathbf{X}_M^T \mathbf{X}_M)$ | $\underline{e}(\mathbf{X})$ | I_1 | I_2 |
|-----|-----|-------------------------------------|-----------------------------|-------|-------|
| 195 | 7 | 2.50559401382184e+66 | 99.94463% | 0 | 0 |
| 196 | 7 | 2.88543051438292e+66 | 99.87926% | 0 | 0 |
| 197 | 7 | 3.3296559952277e+66 | 99.89721% | 0 | 0 |
| 198 | 7 | 3.85377074447653e+66 | 99.92552% | 0 | 0 |
| 199 | 7 | 4.43696061864063e+66 | 99.88295% | 0 | 0 |
| 200 | 7 | 5.10749793017959e+66 | 99.82816% | 0 | 0 |
| 201 | 7 | 5.91761714990989e+66 | 99.86885% | 0 | 0 |
| 202 | 7 | 6.87459220687474e+66 | 99.91884% | 0 | 0 |
| 203 | 7 | 7.93460696696227e+66 | 99.89559% | 0 | 0 |
| 204 | 7 | 9.19706504108042e+66 | 99.87601% | 0 | 0 |
| 205 | 7 | 1.06673644542253e+67 | 99.92901% | 0 | 0 |
| 206 | 7 | 1.23994537039926e+67 | 99.98955% | 0 | 0 |
| 207 | 7 | 1.44087534427919e+67 | 100% | 0 | 1 |
| 208 | 7 | 1.67431324921828e+67 | 100% | 1 | 1 |
| 209 | 7 | 1.90775115415737e+67 | 100% | 1 | 0 |
| 210 | 7 | 2.17369699015561e+67 | 100% | 1 | 0 |
| 211 | 7 | 2.47631844068215e+67 | 99.95199% | 0 | 0 |
| 212 | 7 | 2.82284903881742e+67 | 99.89642% | 0 | 0 |
| 213 | 7 | 3.22338818040641e+67 | 99.91184% | 0 | 0 |
| 214 | 7 | 3.69033305766753e+67 | 99.93627% | 0 | 0 |
| 215 | 7 | 4.20575213212511e+67 | 99.89919% | 0 | 0 |
| 216 | 7 | 4.79241652990297e+67 | 99.85239% | 0 | 0 |
| 217 | 7 | 5.49235437689408e+67 | 99.88801% | 0 | 0 |
| 218 | 7 | 6.30779565955428e+67 | 99.93096% | 0 | 0 |
| 219 | 7 | 7.20440168535099e+67 | 99.91055% | 0 | 0 |

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| N | k | $\det(\mathbf{X}_M^T \mathbf{X}_M)$ | $\underline{e}(\mathbf{X})$ | I_1 | I_2 |
|-----|-----|-------------------------------------|-----------------------------|-------|-------|
| 220 | 7 | 8.25810227449791e+67 | 99.89384% | 0 | 0 |
| 221 | 7 | 9.47167984594683e+67 | 99.93962% | 0 | 0 |
| 222 | 7 | 1.08831420139003e+68 | 99.99167% | 0 | 0 |
| 223 | 7 | 1.25019893532286e+68 | 100% | 0 | 1 |
| 224 | 7 | 1.43612595647344e+68 | 100% | 1 | 1 |
| 225 | 7 | 1.62205297762401e+68 | 100% | 1 | 0 |
| 226 | 7 | 1.83202228599235e+68 | 100% | 1 | 0 |
| 227 | 7 | 2.06887970333071e+68 | 99.95798% | 0 | 0 |
| 228 | 7 | 2.33770977623033e+68 | 99.91016% | 0 | 0 |
| 229 | 7 | 2.64531286804115e+68 | 99.92355% | 0 | 0 |
| 230 | 7 | 3.00021819597880e+68 | 99.94485% | 0 | 0 |
| 231 | 7 | 3.38926876592111e+68 | 99.91227% | 0 | 0 |
| 232 | 7 | 3.82824769878614e+68 | 99.87184% | 0 | 0 |
| 233 | 7 | 4.34624386638258e+68 | 99.90326% | 0 | 0 |
| 234 | 7 | 4.94266541049189e+68 | 99.94056% | 0 | 0 |
| 235 | 7 | 5.59427720359432e+68 | 99.92252% | 0 | 0 |
| 236 | 7 | 6.35134867527976e+68 | 99.90808% | 0 | 0 |
| 237 | 7 | 7.21493813470437e+68 | 99.94801% | 0 | 0 |
| 238 | 7 | 8.20840321641017e+68 | 99.99324% | 0 | 0 |
| 239 | 7 | 9.3367820952159e+68 | 100% | 0 | 1 |
| 240 | 7 | 1.06200365064067e+69 | 100% | 1 | 1 |
| 241 | 7 | 1.19032909175975e+69 | 100% | 1 | 0 |
| 242 | 7 | 1.33414208611734e+69 | 100% | 1 | 0 |
| 243 | 7 | 1.49514380622790e+69 | 99.96292% | 0 | 0 |
| 244 | 7 | 1.67646248709473e+69 | 99.92134% | 0 | 0 |

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Table 1 – continued from previous page

| N | k | $\det(\mathbf{X}_M^T \mathbf{X}_M)$ | $\underline{e(\mathbf{X})}$ | I_1 | I_2 |
|-----|-----|-------------------------------------|-----------------------------|-------|-------|
| 245 | 7 | 1.88211690245739e+69 | 99.93308% | 0 | 0 |
| 246 | 7 | 2.11725917647815e+69 | 99.95181% | 0 | 0 |
| 247 | 7 | 2.37348030462267e+69 | 99.92297% | 0 | 0 |
| 248 | 7 | 2.66038690380169e+69 | 99.88768% | 0 | 0 |
| 249 | 7 | 2.99565482794315e+69 | 99.91559% | 0 | 0 |
| 250 | 7 | 3.37780437076653e+69 | 99.94829% | 0 | 0 |
