

**$k = 5$  search results discussed in  
*D*-optimal and near *D*-optimal  $2^k$  fractional  
factorial designs of resolution V  
by Bulutoglu and Ryan (2006)**

The table below lists search results for  $k = 5$  factor designs. Column definitions are the number of runs  $N$ , the number of factors  $k$ , determinant of the information matrix  $\det(\mathbf{X}_M^T \mathbf{X}_M)$ , a lower bound for *D*-efficiency  $e(\mathbf{X})$ , an indicator for type 1 optimality  $I_1$ , and an indicator for type 2 optimality  $I_2$ . For  $N \equiv 1$  or  $2 \pmod{16}$ , type 1 optimality is restricted generalized type 1 optimality as studied in Chadjiconstantinidis et al. (1989). For the remaining cases, generalized type 1 optimality is as in Cheng (1980).

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
16	5	1.844674e+019	100.00000%	1	1
17	5	3.689349e+019	100.00000%	1	0
18	5	7.378698e+019	100.00000%	1	0
19	5	1.475740e+020	99.56168%	0	0
20	5	2.951479e+020	95.13658%	0	0
21	5	5.902958e+020	95.76512%	0	0
22	5	1.180592e+021	96.39781%	0	0
23	5	2.361183e+021	96.54397%	0	0
24	5	4.722366e+021	94.28090%	0	0
25	5	9.444733e+021	95.36139%	0	0
26	5	1.888947e+022	96.45426%	0	0
27	5	3.777893e+022	97.03281%	0	0
28	5	7.555786e+022	96.10245%	0	0
29	5	1.511157e+023	97.56194%	0	0
30	5	3.022315e+023	99.04359%	0	0
31	5	6.044629e+023	100.00000%	0	1

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Table 1 – continued from previous page

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
32	5	1.208926e+024	100.00000%	1	1
33	5	1.813389e+024	100.00000%	1	0
34	5	2.720083e+024	100.00000%	1	0
35	5	4.080125e+024	99.49909%	0	0
36	5	6.120187e+024	98.37173%	0	0
37	5	9.180280e+024	98.60404%	0	0
38	5	1.377042e+025	98.83690%	0	0
39	5	2.065563e+025	98.62692%	0	0
40	5	3.098345e+025	97.97959%	0	0
41	5	4.647517e+025	98.40300%	0	0
42	5	6.971275e+025	98.82823%	0	0
43	5	1.045691e+026	98.86179%	0	0
44	5	1.568537e+026	98.57476%	0	0
45	5	2.352805e+026	99.16372%	0	0
46	5	3.529208e+026	99.75618%	0	0
47	5	5.293812e+026	100.00000%	0	1
48	5	7.940718e+026	100.00000%	1	1
49	5	1.058762e+027	100.00000%	1	0
50	5	1.411683e+027	100.00000%	1	0
51	5	1.882244e+027	99.68853%	0	0
52	5	2.509659e+027	99.19107%	0	0
53	5	3.346212e+027	99.31153%	0	0
54	5	4.461616e+027	99.43213%	0	0
55	5	5.948822e+027	99.27660%	0	0
56	5	7.931762e+027	98.97433%	0	0

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Table 1 – continued from previous page

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
57	5	1.057568e+028	99.19955%	0	0
58	5	1.410091e+028	99.42529%	0	0
59	5	1.880121e+028	99.40404%	0	0
60	5	2.506829e+028	99.26452%	0	0
61	5	3.342438e+028	99.58325%	0	0
62	5	4.456584e+028	99.90301%	0	0
63	5	5.942112e+028	100.00000%	0	1
64	5	7.922816e+028	100.00000%	1	1
65	5	9.903520e+028	100.00000%	1	0
66	5	1.237940e+029	100.00000%	1	0
67	5	1.547425e+029	99.79833%	0	0
68	5	1.934281e+029	99.51730%	0	0
69	5	2.417852e+029	99.59093%	0	0
70	5	3.022315e+029	99.66462%	0	0
71	5	3.777893e+029	99.55593%	0	0
72	5	4.722366e+029	99.38080%	0	0
73	5	5.902958e+029	99.52048%	0	0
74	5	7.378698e+029	99.66036%	0	0
75	5	9.223372e+029	99.63419%	0	0
76	5	1.152922e+030	99.55175%	0	0
77	5	1.441152e+030	99.75160%	0	0
78	5	1.801440e+030	99.95186%	0	0
79	5	2.251800e+030	100.00000%	0	1
80	5	2.814750e+030	100.00000%	1	1
81	5	3.377700e+030	100.00000%	1	0

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Table 1 – continued from previous page

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
82	5	4.053240e+030	100.00000%	1	0
83	5	4.863888e+030	99.86029%	0	0
84	5	5.836665e+030	99.67954%	0	0
85	5	7.003998e+030	99.72919%	0	0
86	5	8.404798e+030	99.77887%	0	0
87	5	1.008576e+031	99.70019%	0	0
88	5	1.210291e+031	99.58592%	0	0
89	5	1.452349e+031	99.68099%	0	0
90	5	1.742819e+031	99.77615%	0	0
91	5	2.091383e+031	99.75281%	0	0
92	5	2.509659e+031	99.69838%	0	0
93	5	3.011591e+031	99.83543%	0	0
94	5	3.613909e+031	99.97266%	0	0
95	5	4.336691e+031	100.00000%	0	1
96	5	5.204029e+031	100.00000%	1	1
97	5	6.071367e+031	100.00000%	1	0
98	5	7.083262e+031	100.00000%	1	0
99	5	8.263806e+031	99.89789%	0	0
100	5	9.641107e+031	99.77183%	0	0
101	5	1.124796e+032	99.80757%	0	0
102	5	1.312262e+032	99.84332%	0	0
103	5	1.530972e+032	99.78414%	0	0
104	5	1.786134e+032	99.70370%	0	0
105	5	2.083823e+032	99.77258%	0	0
106	5	2.431127e+032	99.84151%	0	0

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Table 1 – continued from previous page

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
107	5	2.836315e+032	99.82186%	0	0
108	5	3.309034e+032	99.78323%	0	0
109	5	3.860539e+032	99.88307%	0	0
110	5	4.503963e+032	99.98300%	0	0
111	5	5.254623e+032	100.00000%	0	1
112	5	6.130394e+032	100.00000%	1	1
113	5	7.006164e+032	100.00000%	1	0
114	5	8.007045e+032	100.00000%	1	0
115	5	9.150908e+032	99.92224%	0	0
116	5	1.045818e+033	99.82930%	0	0
117	5	1.195221e+033	99.85625%	0	0
118	5	1.365966e+033	99.88322%	0	0
119	5	1.561105e+033	99.83722%	0	0
120	5	1.784119e+033	99.77753%	0	0
121	5	2.038994e+033	99.82973%	0	0
122	5	2.330279e+033	99.88196%	0	0
123	5	2.663175e+033	99.86554%	0	0
124	5	3.043629e+033	99.83672%	0	0
125	5	3.478433e+033	99.91269%	0	0
126	5	3.975352e+033	99.98872%	0	0
127	5	4.543260e+033	100.00000%	0	1
128	5	5.192297e+033	100.00000%	1	1
129	5	5.841334e+033	100.00000%	1	0
130	5	6.571501e+033	100.00000%	1	0
131	5	7.392938e+033	99.93886%	0	0

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Table 1 – continued from previous page

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
132	5	8.317056e+033	99.86750%	0	0
133	5	9.356688e+033	99.88855%	0	0
134	5	1.052627e+034	99.90961%	0	0
135	5	1.184206e+034	99.87290%	0	0
136	5	1.332231e+034	99.82684%	0	0
137	5	1.498760e+034	99.86776%	0	0
138	5	1.686105e+034	99.90870%	0	0
139	5	1.896869e+034	99.89493%	0	0
140	5	2.133977e+034	99.87259%	0	0
141	5	2.400724e+034	99.93234%	0	0
142	5	2.700815e+034	99.99213%	0	0
143	5	3.038417e+034	100.00000%	0	1
144	5	3.418219e+034	100.00000%	1	1
145	5	3.798021e+034	100.00000%	1	0
146	5	4.220023e+034	100.00000%	1	0
147	5	4.688915e+034	99.95069%	0	0
148	5	5.209905e+034	99.89417%	0	0
149	5	5.788784e+034	99.91107%	0	0
150	5	6.431982e+034	99.92797%	0	0
151	5	7.146647e+034	99.89801%	0	0
152	5	7.940718e+034	99.86140%	0	0
153	5	8.823021e+034	99.89434%	0	0
154	5	9.803356e+034	99.92729%	0	0
155	5	1.089262e+035	99.91564%	0	0
156	5	1.210291e+035	99.89782%	0	0

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Table 1 – continued from previous page

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
157	5	1.344768e+035	99.94605%	0	0
158	5	1.494186e+035	99.99429%	0	0
159	5	1.660207e+035	100.00000%	0	1
160	5	1.844674e+035	100.00000%	1	1
161	5	2.029142e+035	100.00000%	1	0
162	5	2.232056e+035	100.00000%	1	0
163	5	2.455262e+035	99.95940%	0	0
164	5	2.700788e+035	99.91353%	0	0
165	5	2.970867e+035	99.92739%	0	0
166	5	3.267953e+035	99.94126%	0	0
167	5	3.594749e+035	99.91636%	0	0
168	5	3.954223e+035	99.88656%	0	0
169	5	4.349646e+035	99.91364%	0	0
170	5	4.784610e+035	99.94074%	0	0
171	5	5.263071e+035	99.93077%	0	0
172	5	5.789379e+035	99.91623%	0	0
173	5	6.368316e+035	99.95597%	0	0
174	5	7.005148e+035	99.99573%	0	0
175	5	7.705663e+035	100.00000%	0	1
176	5	8.476229e+035	100.00000%	1	1
177	5	9.246795e+035	100.00000%	1	0
178	5	1.008741e+036	100.00000%	1	0
179	5	1.100445e+036	99.96600%	0	0
180	5	1.200486e+036	99.92802%	0	0
181	5	1.309621e+036	99.93960%	0	0

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Table 1 – continued from previous page

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
182	5	1.428677e+036	99.95118%	0	0
183	5	1.558557e+036	99.93017%	0	0
184	5	1.700244e+036	99.90544%	0	0
185	5	1.854811e+036	99.92810%	0	0
186	5	2.023431e+036	99.95078%	0	0
187	5	2.207379e+036	99.94217%	0	0
188	5	2.408050e+036	99.93008%	0	0
189	5	2.626963e+036	99.96340%	0	0
190	5	2.865778e+036	99.99672%	0	0
191	5	3.126303e+036	100.00000%	0	1
192	5	3.410513e+036	100.00000%	1	1
193	5	3.694722e+036	100.00000%	1	0
194	5	4.002615e+036	100.00000%	1	0
195	5	4.336167e+036	99.97111%	0	0
196	5	4.697514e+036	99.93916%	0	0
197	5	5.088974e+036	99.94897%	0	0
198	5	5.513055e+036	99.95879%	0	0
199	5	5.972476e+036	99.94082%	0	0
200	5	6.470182e+036	99.91997%	0	0
201	5	7.009364e+036	99.93921%	0	0
202	5	7.593478e+036	99.95847%	0	0
203	5	8.226268e+036	99.95097%	0	0
204	5	8.911790e+036	99.94076%	0	0
205	5	9.654439e+036	99.96909%	0	0
206	5	1.045898e+037	99.99743%	0	0

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Table 1 – continued from previous page

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
207	5	1.133056e+037	100.00000%	0	1
208	5	1.227477e+037	100.00000%	1	1
209	5	1.321898e+037	100.00000%	1	0
210	5	1.423583e+037	100.00000%	1	0
211	5	1.533089e+037	99.97516%	0	0
212	5	1.651019e+037	99.94789%	0	0
213	5	1.778021e+037	99.95632%	0	0
214	5	1.914791e+037	99.96474%	0	0
215	5	2.062083e+037	99.94921%	0	0
216	5	2.220705e+037	99.93139%	0	0
217	5	2.391528e+037	99.94794%	0	0
218	5	2.575492e+037	99.96448%	0	0
219	5	2.773607e+037	99.95791%	0	0
220	5	2.986961e+037	99.94916%	0	0
221	5	3.216727e+037	99.97355%	0	0
222	5	3.464168e+037	99.99795%	0	0
223	5	3.730642e+037	100.00000%	0	1
224	5	4.017615e+037	100.00000%	1	1
225	5	4.304587e+037	100.00000%	1	0
226	5	4.612058e+037	100.00000%	1	0
227	5	4.941490e+037	99.97841%	0	0
228	5	5.294454e+037	99.95487%	0	0
229	5	5.672629e+037	99.96219%	0	0
230	5	6.077817e+037	99.96950%	0	0
231	5	6.511947e+037	99.95593%	0	0

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Table 1 – continued from previous page

$N$	$k$	$\det(\mathbf{X}_M^T \mathbf{X}_M)$	$e(\mathbf{X})$	$I_1$	$I_2$
232	5	6.977086e+037	99.94053%	0	0
233	5	7.475449e+037	99.95491%	0	0
234	5	8.009410e+037	99.96929%	0	0
235	5	8.581511e+037	99.96347%	0	0
236	5	9.194476e+037	99.95590%	0	0
237	5	9.851224e+037	99.97712%	0	0
238	5	1.055488e+038	99.99833%	0	0
239	5	1.130880e+038	100.00000%	0	1
240	5	1.211657e+038	100.00000%	1	1
241	5	1.292435e+038	100.00000%	1	0
242	5	1.378597e+038	100.00000%	1	0
243	5	1.470503e+038	99.98106%	0	0
244	5	1.568537e+038	99.96054%	0	0
245	5	1.673106e+038	99.96695%	0	0
246	5	1.784647e+038	99.97335%	0	0
247	5	1.903623e+038	99.96141%	0	0
248	5	2.030531e+038	99.94796%	0	0
249	5	2.165900e+038	99.96057%	0	0
250	5	2.310293e+038	99.97317%	0	0