Modeling midwave infrared muzzle flash spectra from unsuppressed and flash-suppressed large caliber munitions

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ABSTRACT

Time-resolved infrared spectra of firings from a 152 mm howitzer were acquired over an 1800–6000 cm⁻¹ spectral range using a Fourier-transform spectrometer. The instrument collected primarily at 32 cm⁻¹ spectral and 100 Hz temporal resolutions. Munitions included unsuppressed and chemically flash suppressed propellants. Secondary combustion occurred with unsuppressed propellants resulting in flash emissions lasting ~100 ms and dominated by H₂O and CO₂ spectral structure. Non-combusting plume emissions were one-tenth as intense and approached background levels within 20–40 ms. A low-dimensional phenomenological model was used to reduce the data to temperatures, soot absorbances, and column densities of H₂O, CO₂, CH₄, and CO. The combusting plumes exhibit peak temperatures of ~1400 K, areas of greater than 32 m², low soot emissivity of ~0.04, with nearly all the CO converted to CO₂. The non-combusting plumes exhibit lower temperatures of ~1000 K, areas of ~5 m², soot emissivity of greater than 0.38 and CO as the primary product. Maximum fit residual relative to peak intensity are 14% and 8.9% for combusting and non-combusting plumes, respectively. The model was generalized to account for turbulence-induced variations in the muzzle plumes. Distributions of temperature and concentration in 1–2 spatial regions demonstrate a reduction in maximum residuals by 40%. A two-region model of combusting plumes provides a plausible interpretation as a ~1550 K, optically thick plume core and ~2550 K, thin, surface-layer flame-front. Temperature rate of change was used to characterize timescales and energy release for plume emissions. Heat of combustion was estimated to be ~5 MJ/kg.

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1. Introduction

Muzzle blast and flash signatures are important for gun design operator safety and determining firing location [1,2]. Despite the apparent utility of the signatures, study of muzzle flash has focused on its occurrence and suppression [1–3], and there are only a handful of works on the characteristics of flash signatures [4–7]. No modern spectroscopic studies could be found in the literature. This is surprising considering that remote observation of signatures offers the potential for new academic and practical applications. Temporally-resolved spectral signatures may improve our understanding of the thermochemistry and fluid dynamics of muzzle plumes.

The most significant contributor to muzzle flash – both in terms of size and intensity – is secondary combustion [1]. This is the re-ignition of muzzle gases and particulate matter that have been expelled from the gun barrel. It can only occur after mixing with atmospheric oxygen [2,3,8]. After re-ignition, the flame front quickly envelops the entire muzzle plume, resulting in emissions throughout the visible and infrared. Nearly 99% of the energy...
radiated in muzzle flash is in the infrared, making it an ideal spectral region to characterize for practical use [5]. Even if re-ignition does not occur (such as with the use of flash suppressors) the plume that develops from the muzzle flow contains high concentrations of high-temperature particulate matter that emit continuum radiation throughout the infrared [2,8].

Midwave infrared (MWIR, 1–5 μm) spectral features from high-explosive (HE) fireballs have recently been used to identify emissive area, temperature, and emitter species at specific temperatures and concentrations [9–11]. These models have not been applied or adapted for muzzle flash. Combusting muzzle plumes are similar to fireballs – gaseous fuel and particulate matter burn with entrained atmospheric oxygen in a hot, turbulent mixture – and the work presented here extends the application of the HE model to muzzle flash spectra.

The purpose of this paper is three-fold. First, we present and characterize recent time-resolved spectral measurements of howitzer muzzle flashes obtained during a test designed to develop an understanding of large caliber gun firing signatures. Midwave infrared (1.5–5.5 μm) emissions from secondary-combusting and non-combusting plumes are presented. Second, we explore the utility of a highly-simplified, low-dimensional spectral model for dimensionality reduction and feature extraction. (If this model adequately accounts for the spectral variance in the flash signatures, it will be used to extract features from hundreds of events for follow-on weapons classification work.) Evolving flash temperatures, sizes and emitter concentrations are estimated, and temperature dynamics are examined in terms of combustion heat release. Finally, we consider the limitations of this simple model and improve upon it by accounting for, in an approximate and computationally efficient way, the spatial non-uniformity observed in the muzzle flow field. In concert with these efforts, a reactive-flow computational fluid dynamic simulation of the muzzle flow field has been started [12]. When complete, the CFD results will be used to inform the development of a more realistic, multi-layer spectral plume model so that more representative, physics-based parameters might be extracted.

2. Experimental

Two hundred and one firings of a 152 mm howitzer were conducted during 10–19 October, 2007. The test was instrumented with numerous spectrometers, radiometers, and high-speed imagers spanning the visible and infrared, and gun firings were observed from one of two locations. Only those details pertinent to this study are reported here, and additional details on test execution, instrumentation, and layout can be found in Ref. [13].

The gun fired munitions with three different carbon, hydrogen, nitrogen, and oxygen (CHNO) double-base propellants designated Full Charge (8.85 kg), Charge 1 (7.52 kg), and Charge 2 (4.24 kg). All propellants were under-oxidized. Due to the length of the howitzer barrel, available oxygen is consumed and combustion ceases interior to the gun. Prior to exiting the 152 mm howitzer muzzle, the flow is shock heated when it passes through a brake that splits the flow into one forward and two lateral plumes [1,2,8]. When the hot muzzle effluent turbulently entrains atmospheric oxygen, the gases may re-ignite and combustion can continue to completion. To inhibit secondary combustion, Full Charge and Charge 1 both contained optional chemical flash suppressant (0.28 kg) that consume OH and H combustion radicals [2,3].

Table 1 indicates the relative stoichiometry for each propellant. For complete combustion, all propellant is converted to H2O, CO2, and N2. This neglects non-CHNO species, which account for less than 1% of the propellant composition. In this case, the quantity \( R = \frac{2[H_2O]}{([CO_2] + [CO])} \) is equivalent to the stoichiometric hydrogen-to-carbon (H:C) ratio. The brackets denote number density of the indicated species. If the muzzle plume does not re-ignite and secondary combustion does not occur, the under-oxidized propellant burns only with oxygen available interior to the gun. The products of incomplete combustion are preferentially H2O and CO, with CO2 being formed only if additional oxygen is available [14]. Remaining hydrogen and carbon primarily produce H2 and soot, and negligible CO2 concentrations are assumed. In non-combusting plumes, \( R_d = R \) includes only the primary hydrogen and carbon containing species (i.e. H2 and soot are neglected) oxidized by the oxygen only in the propellant. These theoretical incomplete combustion \( R_v \) values are also shown in Table 1 and represent what might be observed spectroscopically when the muzzle flash does not afterburn.

An ABB-Bomen MR-254 Fourier transform spectrometer (FTS) was located with a side-on view of the gun at a distance of 429 m. The instrument has two channels that were equipped with InGaAs (5800–10,000 cm\(^{-1}\)) and InSb (1800–7800 cm\(^{-1}\)) detectors. The detectors shared a common input aperture with a visible, bore-sight camera used for pointing and focus. Field of view (FOV) was limited by a 75 mrad telescope providing a radial FOV of approximately 16 m at the gun, sufficient to contain the entire muzzle plume whose maximum dimension was 8–10 m (oriented along the firing axis). The plumes’ side-on projected area was typically a maximum of 35–40 m\(^2\) (about 5% of the FOV). The bore-sighted visible camera was used to ensure that the plume was within and under-filled the instrument’s FOV.

The MR-254 is a Michelson-type interferometer in which light is split between two optical paths then recombined to form a constructive and destructive interference pattern. The instrument samples the interference pattern, termed an interferogram. To correct for errors in sampling, each double-sided interferogram was phase corrected using Mertz’s technique with \( n = 182 \) points sampled to either side of the centerburst [15,16]. The corrected interferograms were then apodized with a Hanning function, \( H(x) = \cos^2(\pi x/2kn) \), and Fast-Fourier Transformed to yield raw spectral irradiances, \( E_v(v) \). Complex gain and offset calibration coefficients were used to center and scale raw irradiance data to radiometrically accurate values. Blackbody measurements of six area–temperature pairs were taken several times per day, and
the nearest (in time) was used to obtain the calibration coefficients for each firing. Details of this calibration procedure can be found in Refs. [10,11]. Calibrated MWIR spectral irradiances were converted to apparent spectral intensities via the approximation

$$I_0 = E_0(\lambda) r^2$$

where $r$ is range to source and source area, $A$, represents a small angle (i.e. $r^2 \gg A$). A collection of spectral intensities obtained from successive interferograms form a spectral data cube. A representative spectral data cube is shown in Fig. 1 for a Full Charge firing’s secondary combustion plume.

For each firing, the FTS acquired data at rates of 100 Hz, 82 Hz, and 35 Hz corresponding to nominal spectral resolutions of 32 cm$^{-1}$ ($\Delta v = 15.43$ cm$^{-1}$, sampled with $n = 2048$ points), 16 cm$^{-1}$ ($\Delta v = 7.71$ cm$^{-1}$, $n = 4096$), and 4 cm$^{-1}$ ($\Delta v = 1.93$ cm$^{-1}$, $n = 16384$). The instrument’s sampling period was moderately faster than the event duration. Corruption of the spectra by scene change artifacts (SCAs) was assessed as negligible based on the lack of spectral structure in the imaginary component of the spectrum. While counter-intuitive, FTS has been successfully used to study other rapidly-evolving combustion systems and SCAs have been previously addressed [17,18]. The total numbers of firings, firings acquired at each spectral resolution, and those resulting in usable spectra are indicated in Table 1 for each munitions configuration. The MR-254 did not observe all firings, which accounts for the difference between the total number of firings and those for which various resolution spectra were acquired. Throughout the test the instrument’s signal was amplified or attenuated through a combination of electrical gain and neutral density filters placed in the input aperture in order to maximize the signal-to-noise ratio (SNR) and avoid saturation. In some cases, the collected data were not useable because excessive noise or saturation of the interferogram occurred.

3. Results

3.1. Observed spectra

Peak spectra for both a combusting (unsuppressed) and non-combusting (suppressed) plume are shown in Fig. 2. The spectra for all Full Charge, Charge 1, and Charge 2 munitions were not obviously different – likely because their propellant stoichiometries are similar – and are not distinguished in the work presented here. The plume filled at most 5% of the FOV, and background emissions account for <1% band-integrated intensity in combusting plumes and 5–15% in non-combusting. A background spectrum acquired at local, near-noon is included in Fig. 2 for reference. The background was removed by subtracting the average of several spectra acquired prior to the gun firing.

The muzzle plume’s emissions are detectable in the MWIR regardless of flash, but the magnitude and spectral characteristics are highly dependent on whether combustion occurs. Non-combusting plumes are dominated by continuum radiation throughout much of the MWIR, and visible imagery shows the plumes to be dark and opaque, consistent with large concentrations of soot resulting from under-oxidized propellant [5–7]. Prior studies have found that muzzle exhausts for fuel-rich propellants contain a high concentration of particulates ($10^3$–$10^6$ cm$^{-3}$) that emit continuum radiation [1,8].

If temperatures remain above 900–1000 K after atmospheric oxygen has been turbulent entrained into the muzzle flow, then the plume may re-ignite and propellant combustion by-products can continue to burn to completion [1–3]. Combustion consumes the particulate matter, which was previously found to vaporize near 1000 K [8]. Products of vaporization and continued combustion of muzzle gases are primarily H$_2$O, CO$_2$, and N$_2$ with trace CO, NO, and species containing contaminants such as K, Na, Ca, and Cu [14]. Consumption of the particulates eliminates most of the graybody emissions, and the observed MWIR spectra show highly structured features. This is also apparent in the visible imagery where the once sooty plume becomes transparent; for example, in Fig. 2 the howitzer’s barrel is clearly visible through the combusting plume whereas it was completely obscured in the non-combusting plume imagery.

Band-integrated total intensity is approximately ten times greater for combusting plumes, due to elevated temperatures and larger emissive areas. Hot H$_2$O and CO$_2$ are particularly emissive in the MWIR, and account for a significant fraction of the observed intensity. Previous studies of explosive detonations have demonstrated that MWIR spectra can be used to estimate the H$_2$O and CO$_2$ concentrations in combusting HE fireballs using a low-dimensional radiative transfer model [10,11]. Although the timescales are different – Fig. 1 showed muzzle plume combustion to be complete in less than 100 ms whereas HE fireballs may last 0.5–5 or longer [9,11,19] – the similarity in thermochemistry (if not kinetics), allows the HE model to be used to estimate temperature and concentrations of H$_2$O and CO$_2$ from the muzzle flash spectra. In the case of non-combusting plumes, the model can also be used to estimate temperature for highly graybody emissions.

3.2. Spectral model

The HE model extracts the evolving temperature, emissive area, particulate emissivity, and column densities for H$_2$O, CO$_2$, and CO. This is accomplished by a least-squares fit of the HE model to measured combustion spectra via non-linear regression. Spectra of fireballs resulting from the detonation of high explosive materials are modeled well in the 2500–7000 cm$^{-1}$ range, with typical residuals of less than 5–10% [10,11]. The model is capable of estimating the H:C ratio, a key feature for event classification [10]. We now explore the validity of this model for characterizing muzzle flash spectra.

The model assumes a homogeneous fireball in local thermodynamic equilibrium (LTE) and ignores both scattering and the transmission of background radiation through the source. Under these simplifying assumptions, the source spectral intensity is described as:

$$I_s(v, T, \mu) = A e(v, T, \mu) B(v, T)$$

where $A$ is the projected source area, $e(v, T) = 1 - e^{-a(v, T)}$ the source emissivity, and $B(v, T)$ is the blackbody (Planckian) spectral radiance.

Under the ideal model assumptions, $A$ represents the projected area of the fireball. It is simply a scale factor and will account for...
other effects such as radiometric calibration errors and uniform attenuation by particulates. Rather than fix the area to that observed in visible imagery, it was used as a fit parameter to enable the model to most effectively account for the observed variations in spectral intensity.

The absorbance, \( \alpha \), depends on the species-dependent absorption cross-sections, \( \sigma_i(v) \), column densities, \( \mu_i = n_i l \), and soot absorbance \( \alpha_s \):

\[
\alpha(v; T) = \alpha_s + \sum_i \sigma_i(v; T) n_i l
\]

where \( l \) is the optical path length, \( i \) the index for \( \text{H}_2\text{O}, \text{CO}_2, \text{CH}_4, \) and \( \text{CO} \) species, and \( n_i \) is the concentration of species \( i \).

Generally, the absorption coefficient of soot is defined by the particulate’s wavelength-dependent complex index of refraction, which is sensitive to both the chemical composition and porosity, and to a lesser extent, the soot particle size distribution [20]. In some combustion systems, the wavelength dependence of \( \alpha_s \) can be empirically modeled as \( \alpha_s \sim \lambda^{-a} \) where typically \( 0.7 < a < 2.2 \). In this work, we ignore the wavelength dependence of \( \alpha_s \). This pragmatic, simplifying assumption was made to reduce the number of model parameters to improve speed and convergence of the non-linear fits. (For a few non-combusting plumes, soot absorbance was modeled as \( \alpha_s = \alpha_{s,0} e^{-a \lambda} \) and led to minimal improvements in fit quality. The additional degree of freedom via the \( a \) parameter was not statistically justified and led to unphysical values near 0.2.)

Molecular absorption cross-sections \( \sigma_i \) were computed for \( \text{CO}_2, \text{CO}, \text{CH}_4 \) and \( \text{H}_2\text{O} \) using the Line-by-Line Radiative Transfer Model (LBLRTM) [21]. The high-temperature extension (HITEMP) to the HITRAN spectroscopic database was used to model \( \text{CO}_2, \text{H}_2\text{O} \) and \( \text{CO} \) emission lines [22]. A high-temperature database for methane (\( \text{CH}_4 \)) is not currently available in a format suitable for LBLRTM, so lines from HITRAN were used in this work. These spectroscopic databases include information to compute both the strength and shape of emission lines. For all lines, a Voigt profile is computed and accounts for both Doppler-broadening (Gaussian) and pressure-broadening (Lorentzian) effects.\(^3\) The cross-sections include the Boltzmann factor for relative populations in each internal state, so that \( n_i \) represents the total concentration of the \( i \)th species.

Details of the calculation are provided in references [10,11]. The temperatures, areas, soot emissivity, and species concentrations are determined as a function of time from evolving spectra.

Intensities obtain via Eqs. (1) and (2) were sampled at \( \Delta v = 0.0025 \text{ cm}^{-1} \), the resolution at which the cross-sections were calculated in LBLRTM. For comparison to the MR-254 spectra, instrument responses were calculated by propagating source intensities through atmosphere and convolving with the instrument lineshape (ILS):

\[
I_m(v) = \int_{-\infty}^{\infty} \tau(v') I(v') \text{ILS}(v - v') dv' = F^{-1} \{ \tau(v) I(v) \} H(x)
\]

Atmospheric attenuation, \( \tau(v) \), for the 429 m horizontal path was calculated using LBLRTM with the HITRAN database at \( \Delta v = 0.0025 \text{ cm}^{-1} \) resolution for the median atmospheric pressure (90,200 Pa), temperature (15.6 °C), and relative humidity (37%) recorded during the test. Because the MR-254 is an interferometer, the convolution was implemented as the right-hand side of Eq. (3). The inverse Fast-Fourier Transform, \( F^{-1} \{ \} \), converted the attenuated, high-resolution source intensity to an interferogram that was then apodized with the Hanning function and Fourier Transformed, \( F \{ \} \), back to a spectrum. Resolution in a FTS is defined by the maximum optical path difference (MOPD) between mirrors (\( \Delta v = 1/x_m \)). By truncating the interferogram at \( x_m = 0.032 \text{ cm}, 0.065 \text{ cm}, \) or 0.259 cm, model intensities were obtained at the nominal 32 cm\(^{-1}, 16 \text{ cm}^{-1}, \) or 4 cm\(^{-1} \) spectral resolutions. \( x_m \) is based on actual MOPD recorded during the test and does not exactly match the nominal resolutions.

\(^3\) CO\(_2\) line profiles are sensitive to line mixing and duration-of-collision effects, which result in non-Lorentzian pressure-broadened line shapes, and LBLRTM is capable of accounting for these effects at atmospheric temperatures. Unfortunately, the authors are unaware of work in which these effects have been measured and accurately modeled at combustion temperatures, so the Voigt approximation is used in this work.
3.3. Plume dynamics

The model was fit to each spectrum in a data cube to obtain the time-dependent fit parameters. Area, temperature, soot absorbance, and concentration of H2O, CO2, and CO were varied to minimize root-mean-square (RMS) residuals between observed and simulated spectra. CH4 was included in a subset of spectral fits to assess improvement to fit quality. Fits were performed using a non-linear optimization simplex search method [23]. Representative fits to the peak spectra of combusting and non-combusting plumes are shown in Fig. 3. When significant soot is present, a strong continuum is observed. Residuals between the model and data as a fraction of RMS intensity are indicated in the bottom panels.

Plume dynamics

Fig. 3. HE model (Im, ---) and observed (In, •••) spectral intensities are shown for the peak band-integrated intensity of a combusting (left) and non-combusting (right) plume. Planckian emissions (Io, •••) at equivalent temperatures are shown for reference. Residuals between the model and data as a fraction of RMS intensity are indicated in the bottom panels.

Table 2

<table>
<thead>
<tr>
<th>Fit parameter</th>
<th>Combusting</th>
<th>Non-combusting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area, A (m²)</td>
<td>27.3 ± 1.3</td>
<td>3.64 ± 0.05</td>
</tr>
<tr>
<td>Optical path, I = A¹⁻² (m)</td>
<td>5.22 ± 0.40</td>
<td>1.91 ± 0.23</td>
</tr>
<tr>
<td>Temperature, T (K)</td>
<td>1474 ± 27</td>
<td>1160 ± 2</td>
</tr>
<tr>
<td>Soot absorbance, εa</td>
<td>0.038 ± 0.006</td>
<td>0.49 ± 0.01</td>
</tr>
<tr>
<td>Emissivity, ε= (1 − e⁻ᵃ)</td>
<td>0.038 ± 0.006</td>
<td>0.38 ± 0.01</td>
</tr>
<tr>
<td>H₂O column density, μ (cm⁻²)</td>
<td>4.5 ± 0.1 × 10²⁰</td>
<td>1.28 ± 0.02 × 10²⁰</td>
</tr>
<tr>
<td>Concentration, N = μ/l (cm⁻¹)</td>
<td>4.4 ± 1.1 × 10¹⁷</td>
<td>6.7 ± 0.1 × 10¹⁷</td>
</tr>
<tr>
<td>Gas mixing fraction (%)</td>
<td>13.3 ± 0.4</td>
<td>44.4 ± 3.7</td>
</tr>
<tr>
<td>CO₂ column density, μ (cm⁻²)</td>
<td>10 ± 1 × 10²⁰</td>
<td>0.35 ± 0.14 × 10²⁰</td>
</tr>
<tr>
<td>Concentration, N = μ/l (cm⁻¹)</td>
<td>2.8 ± 0.7 × 10¹⁸</td>
<td>1.8 ± 0.7 × 10¹⁷</td>
</tr>
<tr>
<td>Gas mixing fraction (%)</td>
<td>84.0 ± 1.8</td>
<td>8.7 ± 18.0</td>
</tr>
<tr>
<td>CO column density, μ (cm⁻²)</td>
<td>1.3 ± 0.4 × 10¹⁹</td>
<td>11.5 ± 1.3 × 10¹⁹</td>
</tr>
<tr>
<td>Concentration, N = μ/l (cm⁻¹)</td>
<td>5.0 ± 0.8 × 10¹⁰</td>
<td>60 ± 7 × 10¹⁰</td>
</tr>
<tr>
<td>Gas mixing fraction (%)</td>
<td>2.7 ± 2.2</td>
<td>46.9 ± 45.0</td>
</tr>
<tr>
<td>CH₄ column density, μ (cm⁻²)</td>
<td>4.5 ± 0.1 × 10²⁰</td>
<td>1.28 ± 0.02 × 10²⁰</td>
</tr>
<tr>
<td>Concentration, N = μ/l (cm⁻¹)</td>
<td>N/A</td>
<td>(5.2 ± 1.1) × 10¹⁸</td>
</tr>
<tr>
<td>Gas mixing fraction</td>
<td>46.9 ± 45.0</td>
<td></td>
</tr>
</tbody>
</table>

Methane concentrations were not estimated for all time steps.
Spectral modeling suggests non-combusting plume temperatures are initially near $T_0 = 850$–1050 K and quickly cool from expansion and continued entrainment of cold atmosphere. As temperature falls, emissions decrease rapidly and approach to within 5% of background by 30–40 ms after gun firing: model parameters beyond this time are unreliable due to lack of signal. A decrease in soot emissivity with time is observed but is consistently higher than in combusting plumes. Carbon-monoxide concentrations are also higher in non-combusting plumes, consistent with incomplete combustion of the fuel-rich propellant. Alternatively, if a plume reignites, combustion raises the average temperature to near $T_0 = 1200$–1600 K. Elevated temperatures vaporize soot particulates and provide reaction pathways responsible for large increases in concentrations of H$_2$O and CO$_2$. Burning of soot is indicated by an abrupt decrease in soot emissivity immediately after re-ignition ($t = 10$–20 ms). Sustained burning maintains strong emissions beyond 100 ms even as temperature decreases while fuel is consumed and additional cold air is entrained.

Fig. 4 also shows that the hydrogen-to-carbon ratios (derived from model parameter concentrations) for both plumes are lower than the theoretical values for complete combustion of the propellants. The model may overestimate CO$_2$ concentration and contribute to systematic bias of the H:C ratio. Observed spectral emissions in the 2000–2250 cm$^{-1}$ region are disproportionately more intense than allowed by a single temperature, equilibrium radiation distribution $B(v; T)$. This phenomenon was also observed in HE fireballs and has not yet been explained [10]. It may be the result of non-equilibrium emissions from CO$_2$ or CO – both of which emit due to fundamental vibrational modes in this region – or a distribution of temperatures across the fireball.

Similarly, both H$_2$O and CO$_2$ have a number of overlapping combination and resonance states that are thermally populated at combustion temperatures and emit strongly from 4500 to 5200 cm$^{-1}$. However, no single temperature mixture of H$_2$O and CO$_2$ reproduces the smooth, broad emissions from 4500 to 4850 cm$^{-1}$ (red shoulder) without overemphasizing the 4850–5200 cm$^{-1}$ region (blue shoulder) and introducing an unobserved spike near 4900 cm$^{-1}$. In fitting the model to data, the non-linear optimization obtained an overall lower RMS error by increasing CO$_2$ concentration to reproduce emissions near 2000–2250 cm$^{-1}$ and 4500–4850 cm$^{-1}$. Consequences include overestimation of the amount of CO$_2$ (resulting in a lower H:C ratio) and introduction of systematic residuals throughout the spectrum.

These results indicate that the model does not completely treat phenomenology of the plume. Improvement of the model may require (1) generalizing to a distribution of temperatures and species concentrations across the plume; (2) treating radiative transfer through multiple non-uniform plume layers along the observation path; (3) incorporating an appropriate frequency-dependent soot emissivity; (4) including additional species whose emissions are inadequately compensated for by H$_2$O, CO$_2$, CH$_4$, and CO.

4. Discussion

4.1. Spectral model with spatial variations

To explore one approach for increasing model fidelity, the uniformity assumption of Eq. (1) was relaxed to allow for a distribution of temperatures and H$_2$O, CO$_2$, and CO concentrations across the plume. The distributions are introduced in an attempt to account for the effects of turbulence-induced variations observed in muzzle plumes [1–3]. Radiative transfer is approximated as before, except Eq. (1) is generalized to allow for a distribution of temperatures and concentrations:

$$
\phi(v) = \int_{-\infty}^{\infty} \int_{0}^{\infty} P(T - T_0, \mu) \, l_i(v; T, \mu) \, dT d\mu
$$

(4)

Skewed normal and log-normal distribution functions are employed for the temperature and concentrations, respectively:

$$
P(T, \mu) \equiv C \left( \frac{e^{-[(T - T_0)/\sigma]^2}}{1 + e^{2(T - T_0)/\sigma}} \right) \left( \frac{e^{-(\log(\mu)/\sigma)^2}}{1 + e^{2(\log(\mu) - \mu)/\sigma}} \right)
$$

(5)

Eq. (5) is separable in $T$ and $\mu$, and is unit-area normalized with constant $C$. The distributions for each species $i$ are assumed to have the same spread and skew. The function was chosen because-adjustment of the mean $T_0$, spread $(\Delta T, \Delta \mu)$, and skew $(\gamma_7, \gamma_{10})$ parameters allows for a broad range of continuous and efficiently computable distributions. It should be evident that Eq. (4) is a generalization of Eq. (1), and the two become equivalent as the distribution function approaches a Dirac delta function, $P \rightarrow \delta$. 

Fig. 4. Temporal dependence of HE model parameters when fit to combusting (solid) and non-combusting (dashed) plumes. The values corresponding to peak intensity are indicated (○) and $t = 0$ occurs at gun firing. Left, top to bottom: band-integrated intensity, area, temperature, and soot emissivity. Right, top to bottom: column densities of H$_2$O, CO$_2$, and CO, and H:C ratio. Theoretical H:C ratios for combusting (***), non-combusting (••••), and non-combusting (−−−−) plumes are shown.
In the optically thin limit (or for species each from independent locations), the sum over species in Eq. (2) can be replaced by a sum of the intensities:

\[ \Phi(v) \approx \sum \int \frac{P(T - T_0, \mu/\mu_0) l_i(v; T, \mu) dT d\mu}{l_0} \]  

(6)

The approximation of Eq. (6) is not satisfied for the plume's optically thick conditions, but rather employed for convenience in the computation.

Eqs. (5) and (6) and the spatially-uniform HE model were fit to the peak spectra of a combusting plume, and the results are compared in Fig. 5. When distributions were permitted, residuals demonstrated a 40% reduction in maximum error and 21% reduction in RMS error \( (e = 21.5\% \rightarrow e = 16.9\%) \). For comparison, RMS residuals of the non-distribution, HE model when applied to explosive fireballs were typically 5–10%. This contrast suggests that there is a phenomenological difference between HE fireballs and muzzle plumes that is not explained in the model of Eq. (1) but may be partially mitigated by the use of non-uniformity in Eq. (4).

The best-fit \( \text{H}_2\text{O} \) and temperature distribution functions are also depicted in Fig. 5. \( \text{CO}_2 \) and \( \text{CO} \) follow the same distributions but have different concentrations, provided in Table 3 with all fit parameters. The concentration distribution is skewed by \( \log_{10} \mu_l/\mu_0 \), and it spans a range nearly equal to the spread \( A_{\mu} \approx 0.2 \) (when measured by its full width at half maximum, FWHM). The temperature distribution is biased by \( \psi_T \approx 18 \) to lower temperatures, and the FWHM is also nearly equal to the spread, \( A_T \approx 130 \). Uncertainties in all fit parameters are increased by the use of the generalized model. This is likely due to increased parameterization (6 → 10 fit parameters). Relative uncertainties indicate an insensitivity to skewed and spread. This may indicate that simply using a distribution, regardless of it having a well-defined shape, provides improvement to the model.

The distribution model was also fit to the peak spectrum of a non-combusting plume. Negligible improvement (<4%) was obtained. This is possibly because plumes are sooty and optically thick when they do not combust, and their continuum emissions are insensitive to relatively small variations in concentrations and temperature. Because residuals are not significantly improved and distributions increase parameterization, the HE model is preferred when modeling non-combusting plumes.

### 4.2. Multiple spatially varying regions

![Fig. 5. HE and distribution model fits to a combusting plume. Upper panel: distributions of temperature (—) and \( \text{H}_2\text{O} \) column density (—) centered at \( T_0 = 1472 \text{ K} \) and \( \mu_0 = \mu_{\text{H}_2\text{O}} = 4.7 \times 10^{20} \text{ cm}^{-2} \). \( \text{CO}_2 \) and \( \text{CO} \) have the same column density distribution centered at \( \mu_{\text{CO}_2} = 1.6 \times 10^{20} \text{ cm}^{-2} \) and \( \mu_{\text{CO}} = 8.9 \times 10^{19} \text{ cm}^{-2} \). Lower panel: Residuals between observed spectral intensity and distribution (—) and HE (—) models. RMS error is reduced from \( e = 21.5\% \) to \( e = 16.9\% \) when temperature and concentrations vary spatially.](image)

The 152 mm howitzer is equipped with a muzzle brake that splits the muzzle flow into forward and lateral plumes. This was shown in Fig. 2, from which it is apparent that the plumes are composed of multiple distinct spatial regions. These may be characterized a number of ways. The forward and lateral plumes are two distinct spatial regions apparent to the observer. It may be possible that the each plume has sufficiently different temperatures or concentrations that a model with only a single region – even one containing distributions – cannot reproduce their spectral emissions. The flow may be further divided into sub-regions. For example, each plume consists of thin outer layers in contact with atmosphere, and these layers surround plume cores of \( \sim 3 \text{ m depth} \) [13,24]. The outer layers may be significantly hotter than the core if burning occurs at the surface where atmospheric oxygen is turbulently entrained; likewise core regions may be under-oxidized and consequently have higher concentrations of carbon monoxide. The example is not given as the justification for multiple regions; rather it exemplifies the type of phenomenology that may require a multi-region model.

A multi-region model is obtained as the summation of several source intensities calculated from Eq. (6). Total at-source intensity is composed of emissions from more than one distribution of spectral radiances, each weighted by its region’s emissive area. Note that this formulation treats each region as spatially isolated from the perspective of the observer, e.g. forward and lateral plumes, or core and surface layers viewed at the plume’s edge. Radiative

### Table 3

Model parameters and relative RMS error for the peak intensity spectrum of a combusting muzzle plume.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>HE</th>
<th>1-Region</th>
<th>2-Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (m²)</td>
<td>32.3 ± 4.5</td>
<td>21.6 ± 5.0</td>
<td>20.7 ± 3.9</td>
</tr>
<tr>
<td>( T_0 ) (K)</td>
<td>1389 ± 67</td>
<td>1472 ± 98</td>
<td>1548 ± 151</td>
</tr>
<tr>
<td>( \Delta T ) (K)</td>
<td>130 ± 197</td>
<td>104 ± 142</td>
<td>135 ± 157</td>
</tr>
<tr>
<td>( \mu_{\text{H}_2\text{O}} ) (10²⁰ cm⁻²)</td>
<td>4.71 ± 1.54</td>
<td>4.52 ± 1.19</td>
<td>(1.1 ± 0.8) × 10⁻²</td>
</tr>
<tr>
<td>( \mu_{\text{CO}_2} ) (10²⁰ cm⁻²)</td>
<td>15.78 ± 5.03</td>
<td>10.55 ± 2.77</td>
<td>(8.5 ± 6.1) × 10⁻⁵</td>
</tr>
<tr>
<td>( \mu_{\text{CO}} ) (10²⁰ cm⁻²)</td>
<td>8.91 ± 8.28</td>
<td>0.57 ± 0.64</td>
<td>(1.0 ± 2.7) × 10⁻⁰</td>
</tr>
<tr>
<td>( \Delta \mu_l )</td>
<td>0.21 ± 0.50</td>
<td>0.22 ± 0.34</td>
<td>1.69 ± 0.88</td>
</tr>
<tr>
<td>( \mu_{l} ) (10⁻²)</td>
<td>0.01 ± 0.00</td>
<td>-17.9 ± 22.6</td>
<td>-19.7 ± 23.6</td>
</tr>
<tr>
<td>( R )</td>
<td>0.31</td>
<td>0.57</td>
<td>0.85</td>
</tr>
<tr>
<td>( e ) (%)</td>
<td>21.5</td>
<td>16.9</td>
<td>13.8</td>
</tr>
</tbody>
</table>
transfer through multiple layers at increasing depths from the observer is not treated here.

Fig. 6 depicts the distribution of a two-region model for a combusting plume that reduces RMS error to $e = 13.8\%$. Fit parameters that characterize the distributions of temperatures and concentrations in both regions are given in Table 3. The previous description of a plume comprised of a core and surface layer is a plausible explanation of the parameters. The core is similar to the one-region distribution model (e.g. $A = 21 \text{ m}^2$, $T_0 = 1500 \text{ K}$, etc.) and emissions from this region account for 70% of the observed intensity. These emissions compose the basic structure of the spectrum and include nearly all of the blackbody radiation. Non-distribution, HE model parameters are all located within the distributions of this core region. With the exception of CO column density and emissive area, the mean values of both models are nearly equivalent to within statistical uncertainty.

The second surface layer region is characterized by very low column densities of all species, indicating optical thinness. Mean temperature at $T_0 = 2552 \text{ K}$ is consistent with measured secondary combustion temperatures in excess of 2000–3000 K [8]. The high temperatures and low concentrations may be attributed to a flame-front at the edge of the plume’s core region. The flame-front could be characterized by significant heat release as atmospheric oxygen interacts with the under-oxidized combustion by-products in a very thin surface layer.

4.3. Temperature dynamics

Strong radiative emissions are observed from secondary combustion and suppressed muzzle plumes on timescales of $\sim 100 \text{ ms}$ and $\sim 20 \text{ ms}$, respectively. The timescale difference is evident in temperature cooling rates. Fig. 7 depicts the temperature data for a combusting and suppressed muzzle plume. Temperatures were extracted from the non-combusting plume using the HE model, and from the combusting plume using the HE and distribution models. Combustion temperatures all have similar temporal behavior but differ in magnitude by 200–300 K. All are plausible; however, the temperature derived from the HE model is more stable (owing to the fewer number of fit parameters) and may be preferable.

Rate of temperature change is a balance between multiple competing processes that contribute or dissipate energy. Temperature as a function of time might be simply approximated as a decaying exponential, $T = T_0 e^{-kt}$. Muzzle plume decay rates of $k = 4–6 \text{ s}^{-1}$ (combusting) and $k = 13–21 \text{ s}^{-1}$ (non-combusting) were estimated from the HE model temperatures. The difference in rates can be explained with the use of a recently developed HE model for temperature change [19]:

$$\frac{dT}{dt} = -a(T^4 - T_{\text{atm}}^4) + b(e^{-\alpha t} - e^{-\beta t})$$  (7)

The first term accounts for radiative cooling, and the second term represents the rates of turbulent mixing and reduction in fuel concentration. The coefficients $a$–$d$ are empirically determined, and $b$–$d$ are related to thermodynamic quantities.
Table 4
Temperature rate parameters.

<table>
<thead>
<tr>
<th>Plume</th>
<th>a (10^{-10} s^{-1} K^{-1})</th>
<th>b (10^{4} K s^{-1})</th>
<th>c (s^{-1})</th>
<th>d (s^{-1})</th>
<th>ΔHc (MJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combusting</td>
<td>189.0 ± 10.3</td>
<td>201.2 ± 43.4</td>
<td>54.7 ± 4.0</td>
<td>60.1 ± 45.3</td>
<td>4.9 ± 4.0</td>
</tr>
<tr>
<td>Non-combusting</td>
<td>103.3 ± 1.2</td>
<td>19.1 ± 19.8</td>
<td>74.0 ± 9.9</td>
<td>61.5 ± 34.1</td>
<td>(−0.8 ± 1.2) 0</td>
</tr>
</tbody>
</table>

\[
\Delta H_c \propto c_p b \left(1/c - 1/d\right) \tag{8}
\]

where \(\Delta H_c\) is the specific heat of combustion (J/kg) and \(c_p\) is the specific heat capacity (J/kg K).

Average heat capacity of the 152 mm howitzer propellant is approximately 1900–2000 J/kg K [24]. Assuming stoichiometric mixing and \(\sim 1100\) J/kg K heat capacity of air (average over 300–1400 K), the total specific heat capacity of the plume is estimated to be \(c_p = 1350–1450\) J/kg K.

Table 4 reports parameters for fits of Eq. (7) to the HE temperatures in Fig. 7. The results exemplify the principal difference between combusting and non-combusting plumes, i.e. additional energy is released in combustion as reactants are converted to products of lower internal energy. Specific heat values of 152 mm howitzer munitions were estimated as the heat release from gaseous propellant during combustion divided by the mass of the propellant and sufficient atmosphere for complete, stoichiometric combustion, i.e. \(\Delta H = \Delta H_f(m_p + m_a) \approx 2.54–2.72\) MJ/kg. Heats of combustion from Eq. (8) agree to within statistical uncertainty. Sustained heat release \((\Delta H > 0)\) during combustion of the muzzle plume results in higher temperatures for longer durations, as is evident when comparing the combusting and non-combusting temperature profiles in Fig. 7. The initial increase in temperatures indicates dominance of combusive heating over all cooling effects. As the rate of heating slows (due to consumption of fuel) the plume begins to cool, yet at a slower rate than in non-combusting plumes whose temperature decays monotonically.

For reference, HE fireballs with comparable heats of combustion have been observed with decay rates \(\sim 0.7\) s\(^{-1}\) [19], almost an order of magnitude slower than the combusting muzzle plumes considered in this work. The disparity between these timescales may be due to the relationship between initial and combustion energy releases. In detonations, explosive material is exothermically converted to gaseous products and energy release (heat of detonation) results in a sudden initial temperature increase [19]. This is quickly followed by combustion heat release as the products react with atmospheric oxygen. Detonation and combustion are spatially and temporally nearly coincident, but the ongoing rate of heat release depends on the rate at which oxygen is entrained. During a gun firing, propellant burning occurs interior to the gun and releases energy akin to the heat of detonation. Burning stops once all oxygen is consumed, typically prior to shot exit. Propellant gases cool as they flow from the muzzle and expand outside of the gun [2]. If combustion instability, it is a separate process that occurs after the muzzle plume has developed and begun to mix with atmosphere. The temporal and spatial delay may allow sufficient pre-mixing that burning is not limited by the rate at which oxygen can be entrained, and heat is released much more quickly when the plume is ignited.

### 5. Conclusions

Muzzle plume emission spectra were collected in the midwave infrared \((1800–6000 \text{ cm}^{-1})\) at \(32 \text{ cm}^{-1}\) spectral and 100 Hz temporal resolutions. They represent the highest fidelity spectra of large caliber firing signatures available in the published literature in several decades. Indeed, most prior studies were qualitative in nature, relying on photographic film with analog conversion to spectral intensity plots. The results presented here are the first reported spectra collected with digital instrumentation and amenable to spectral simulation and modeling.

Gun firings of a 152 mm howitzer were observed for unsuppressed and flash-suppressed munitions. Imagery of plumes from all munitions showed \(35–40\) m\(^2\) projected areas from the perspective of the observation site. Plumes from unsuppressed munitions typically combust and are characterized by a maximum emissive area of \(20–35\) m\(^2\), peak temperatures greater than 1200 K, and large concentrations of \(\text{H}_2\text{O}\) and \(\text{CO}_2\). Strong emissions last \(\sim 100\) ms and are dominated by selective radiation. Most flash-suppressed munitions produce non-combusting plumes whose emissive area is \(\sim 5\) m\(^2\) and that are composed primarily of particulate matter and CO at temperatures less than 1100 K. Suppressed spectra are one-tenth as intense, last less than 20–40 ms, and are predominantly continuum and CO emissions with lesser superimposed \(\text{H}_2\text{O}\) and \(\text{CO}_2\) structure. Both plume types have peak spectral emissions in the 2000–2250 cm\(^{-1}\) region that are 50% more intense than in any other region.

A low-dimensional radiative transfer model that characterizes emissions in terms of area, temperature, soot absorbance, and species concentrations of \(\text{H}_2\text{O}, \text{CO}_2,\) and CO was assessed for muzzle flash spectra. The model was recently developed to simulate MWIR combustion emissions from HE detonation fireballs. Results for combusting plumes show temperatures that peak near 1200–1600 K approximately \(\sim 20\) ms after gun firing and cool with exponential decay rates of \(4–6\) s\(^{-1}\). Non-combusting plume temperatures are 850–1050 K and decay monotonically with a much faster 13–21 s\(^{-1}\) rate. Plume ignition results in \(\sim 0.04\) soot emissivity and substantial increases in \(\text{H}_2\text{O}\) and \(\text{CO}_2\), presumably as soot and CO are oxidized during secondary combustion. The non-combusting plume is characterized by \(\sim 0.38\) soot emissivity and CO as the primary constituent. Direct application of the model results in 18–26% RMS fit residuals, and inclusion of \(\text{CH}_4\) lowers residuals to 9–14%. This limits the viability of the model for accurate prediction of muzzle flash spectral features; however, the parameters are plausible of combustion conditions and may be used as classification features that characterize the spectra in low dimensionality.

These results are the first application of a radiative transfer model to MWIR muzzle flash. That spectral features are not adequately reproduced implies that pertinent phenomenology is not described. We investigated generalization of the model by using non-uniform distributions of temperature and species concentrations in the plane of observation. The use of distributions is one of several approaches to improving the model, and the additional degrees of freedom may account for, in an approximate way, the effects of spatially distinct regions and turbulent mixing. In particular, a plausible two-region interpretation of a combusting plume is a \(\sim 1500\) K, optically thick core and \(\sim 2500\) K, thin flame-front at the plume's surface. Fit residuals were reduced to 13–17% RMS error for combusting plumes but negligible improvement was obtained for non-combusting spectra.

The added complexity is not justified for the relatively minor reduction in residuals. Increased parameterization \((6 \rightarrow 10+)\) result in larger parameter uncertainty and possible non-unique solutions. Simulation of muzzle spectra may require additional improvements. These could include a non-graybody treatment of soot emissions, inclusion of additional emitting species, non-equilib-
rium rovibrational distributions, or treating radiative transfer through non-uniformity along the path of observation. Improvements are desired because parameters that describe phenomenology of the plume in low dimensionality may be used to characterize the source of firing emissions. Without such parameters, distinguishing between different firing signatures relies on empirical features that are not always readily understood.

Modeling of temperature rate of change with time demonstrated greater success. Suppressed plume temperatures decay to 500–800 K within ~20 ms, whereas to reach the same temperature requires ~100 ms or longer in combusting plumes. The difference in timescale results from energy released during combustion. When combustion occurs, the temperature of the plume is initially increased and cooling is slowed. Heat of combustion near ~5 MJ/kg was estimated from the model and agrees to within a factor of two with predicted specific heat values for a stoichiometric plume and complete combustion. For comparison, detonation fireballs cool nearly an order of magnitude more quickly for comparable specific heats of combustion. Fireball burn rates are dependent upon rate of oxygen entrainment, and spatial and temporal separation of initial propellant burning from plume combustion in muzzle plumes may allow pre-mixing that accounts for the difference in rates.

References