Quality Insights: Measurement and quality rationing: an analytical approach

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Abstract: Measurement is as important in quality management as it is in many aspects of human endeavours. Take, for example, a business that is attempting to formulate a yearly budget. This business cannot create a budget out of thin air; it must use qualitative and quantitative assessments based on interpretations and equations. The exact methods businesses use to formulate their budgets may vary, but they should all include statistical analysis in order to achieve accuracy and precision. Budget management and quality management are analogous with respect to the application of analytical techniques to make decisions. This paper capitalises on that relationship to apply budget allocation and rationing techniques to quality. This paper presents an analytical approach to measurement and quality rationing in order to meet quality goals. Quality on a scale of measurement is the premise of this paper.

Keywords: measurement; quality management; quality control; capital rationing; budgeting; quality modelling.

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1 Introduction

Statistics, whether in qualitative or quantitative form, is the foundation for accuracy and precision of measurements. This appendix presents a collection of useful statistical definitions, explanations, interpretations, illustrations, examples, formulations, formulas, and equations.

Using the tools and techniques mentioned in this paper, readers can:

- identify and quantify the sources of measurement variability
- assess the effect of the measurement system variability on process variability
- discover opportunities for measurement system and total process variability improvement.

Measurement pervades everything we do. This applies to technical, management, and social activities and requirements. Even in ordinary situations, such as human leisure, the importance of measurement comes to the surface. How much, how far, how good, how fast, and how long are typical connotations of measurement. Throughout history, humans have strived to come up with better tools, techniques, and instruments for measurement. From the very ancient times to the present fast-paced society, our search for more precise, more convenient, and more accessible measuring devices has led to new developments over the years. This pursuit of better measurements is particularly amenable to the management of quality in products and services. Because it is not cost-effective to over-design quality into everything for the sake of providing a safeguard, we must find analytical ways to determine when quality is enough. In other words, what level of quality is achievable, practical, and acceptable for the mission at hand? This paper uses the analytical approach of capital rationing to suggest how quality rationing can be achieved in products and services. Any resource (i.e., capital) needed to impact incremental quality onto a product is the same resource that is needed in other aspects of the business enterprise. Thus, the analogy of capital rationing of quality is applicable.

2 Measurement scales for quality

Every decision requires data collection, measurement, and analysis. In practice, we encounter different types of measurement scales depending on the particular items of interest. Data may need to be collected on decision factors, costs, performance levels, outputs, and so on. The different types of data measurement scales that are applicable for quality assessment include nominal scale, ordinal scale, interval scale, and ratio scale.

Nominal scale is the lowest level of measurement scales. It classifies items into categories. The categories are mutually exclusive and collectively exhaustive. That is, the categories do not overlap and they cover all possible categories of the characteristics being observed. For example, in the analysis of the critical path in a project network, each job is classified as either critical or not critical. Gender, type of industry, job classification, and colour are examples of measurements on a nominal scale.

Ordinal scale is distinguished from a nominal scale by the property of order among the categories. An example is the process of prioritising project tasks for resource allocation. We know that first is above second, but we do not know how far above. Similarly, we know that better is preferred to good, but we do not know by how much. In quality control, the ABC classification of items based on the Pareto distribution is an example of a measurement on an ordinal scale.

Interval scale is distinguished from an ordinal scale by having equal intervals between the units of measurement. The assignment of priority ratings to project objectives on a scale of 0 to 10 is an example of a measurement on an interval scale. Even though an objective may have a priority rating of zero, it does not mean that the objective has absolutely no significance to the project team. Similarly, the scoring of zero on an examination does not imply that a student knows absolutely nothing about the materials covered by the examination. Temperature is a good example of an item that is measured on an interval scale. Even though there is a zero point on the temperature scale, it is an arbitrary relative measure. Other examples of interval scale are IQ measurements and aptitude ratings.

Ratio scale has the same properties of an interval scale, but with a true zero point. For example, an estimate of zero time unit for the duration of a task is a ratio scale measurement. Other examples of items measured on a ratio scale are cost, time, volume, length, height, weight, and inventory level. Many of the items measured in engineering systems will be on a ratio scale.

An important aspect of measurement involves the classification scheme used. Most systems will have both quantitative and qualitative data. Quantitative data require that we describe the characteristics of the items being studied numerically. Qualitative data, on the other hand, are associated with attributes that are not measured numerically. Most items measured on the nominal and ordinal scales will normally be classified into the qualitative data category while those measured on the interval and ratio scales will normally be classified into the quantitative data category. The implication for engineering system control is that qualitative data can lead to bias in the control mechanism because qualitative data are subject to the personal views and interpretations of the person using the data. As much as possible, data for quality management and control should be based on a quantitative measurement. Figure 1 shows the basic measurement characteristics for quality assessment. Precision, accuracy, correlation, stability, and linearity are essential for the purpose of determining when quality is adequately aligned and sufficient for the specific purpose of interest. Figure 2 illustrates a measure of accuracy bias, in which the following equations apply:

$$\mu_{total} = \mu_{product} + \mu_{measurement\ system}$$
$$\sigma^2_{total} = \sigma^2_{product} + \sigma^2_{measurement}$$

Bias is the difference between the observed average value of measurements and the true value. The true value is an accepted, traceable reference standard. The accuracy of the measurement system is determined by conducting a bias study. Figure 3 shows graphical representation of linearity. Linearity is a measure of the difference in accuracy or precision over the range of measurement instrument capability.

Figure 1 Basic measurement characteristics for quality assessment

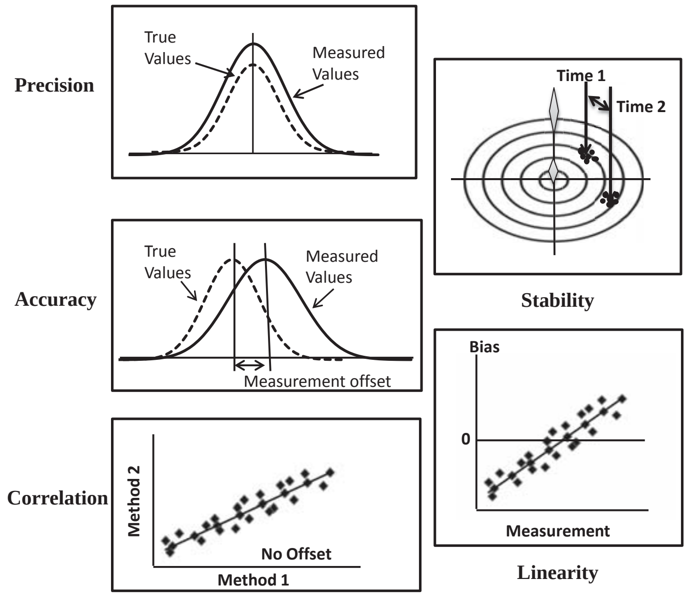


Figure 2 Measure of accuracy bias

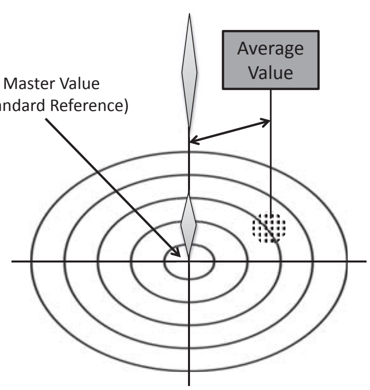
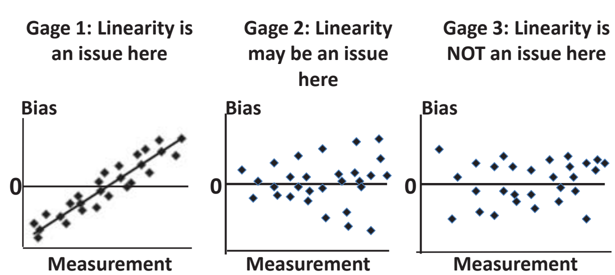


Figure 3 Assessment of linearity (see online version for colours)



3 Quality capital allocation

For the purpose of this section, consider quality design as a budgeting exercise. Budgeting involves sharing limited resources among several project groups or functions contained in a project. A budget *analysis* can serve as any of the following:

- a plan for resources expenditure
- a product selection criterion
- a projection of quality policy
- a basis for quality control
- a performance measure for the organisation
- a standardisation of resource allocation
- an incentive for quality improvement.

Top-down budgeting involves collecting data from upper-level sources such as top and middle managers. The figures supplied by the managers may come from their personal judgment, past experience, or past data on similar project activities. The cost estimates are passed to lower-level managers, who then break the estimates down into specific work components within the project. These estimates may, in turn, be given to line managers, supervisors, and lead workers to continue the process until individual activity costs are obtained. Top management provides the global budget, while the functional-level worker provides specific budget requirements for project items.

In this method, elemental activities and their schedules, descriptions, and labour skill requirements are used to construct detailed budget requests. Line workers familiar with specific activities are asked to provide cost estimates. Estimates are made for each activity in terms of labour time, materials, and machine time. The estimates are then converted to an appropriate cost basis. The dollar estimates are combined into composite budgets at each successive level up the budgeting hierarchy. If estimate discrepancies develop, they can be resolved through the intervention of senior management, middle management, functional managers, project manager, accountants, or standard cost consultants.

Elemental budgets may be developed on the basis of the timed progress of each part of the project. When all the individual estimates are gathered, a composite budget can be developed. Such analytical tools as learning curve analysis, work sampling, and statistical estimation may be employed in the cost estimation and budgeting processes.

4 Mathematical formulation of quality capital allocation

Capital rationing involves selecting a combination of projects that will optimise the return on investment (Badiru et al., 2012; Sieger et al., 2000). A mathematical formulation of the capital (quality) budgeting problem is presented below:

$$\text{Maximise } z = \sum_{i=1}^{n} v_i x_i$$

$$\text{Subject to } \sum_{i=1}^{n} c_i x_i \le B$$

$$x_i = 0, 1; \quad i = 1, \dots, n$$

where

n number of product characteristics

v_i measure of performance for the product characteristic i

c_i cost of product characteristics i

x_i indicator variable for product characteristic i

B budget (quality) availability level.

A solution of the above model will indicate what product characteristic levels should be selected in combination with other product characteristics. The example that follows illustrates a quality rationing problem.

5 Quality rationing model

Planning a portfolio of products is essential in resource-limited production system. The capital-rationing example presented here demonstrates how to determine the optimal combination of product quality investments so as to maximise total return on investment (i.e., product quality value). Let us think of each quality level option as a specific project option. Suppose a statistical analyst is given N projects, X_1, X_2, X_3,…,X_N, with the requirement to determine the level of investment in each project so that total investment return is maximised subject to a specified limit on available budget. The projects are not mutually exclusive.

The investment in each project starts at a base level b_i (i = 1, 2,…,N) and increases by variable increments k_{ij} (j = 1, 2, 3,…,K_i), where K_i is the number of increments used for project i. Consequently, the level of investment in project X_i is defined as follows:

$$x_i = b_i + \sum_{j=1}^{K_i} k_{ij}$$

where

$$x_i \ge 0, \quad \forall i$$

For most cases, the base investment will be zero. In those cases, we will have $b_i = 0$. In the modelling procedure used for this problem, we have

$$X_i = \begin{cases} 1 & \text{if the investment in project is greater than zero} \\ 0 & \text{otherwise} \end{cases}$$

and

$$Y_{ij} = \begin{cases} 1 & \text{if } j^{\text{th}} \text{ increment of alternative } i \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

The variable x_i is the actual level of investment in project i, while X_i is an indicator variable indicating whether or not project i is one of the projects selected for investment. Similarly, k_{ij} is the actual magnitude of the j^{th} increment while Y_{ij} is an indicator variable that indicates whether or not the j^{th} increment is used for project i. The maximum possible investment in each project is defined as M_i, such that

$$b_i \le x_i \le M_i.$$

There is a specified limit, B, on the total budget available to invest, such that

$$\sum_i x_i \le B.$$

There is a known relationship between the level of investment, x_i, in each project and the expected return, $R(x_i)$. This relationship will be referred to as the *quality utility function*, $f(.)$, for the project. The utility function may be developed through historical data, regression analysis, and forecasting models. For a given project, the utility function is used to determine the expected return, $R(x_i)$, for a specified level of investment in that project. That is,

$$\begin{aligned} R(x_i) &= f(x_i) \\ &= \sum_{j=1}^{K_i} r_{ij} Y_{ij} \end{aligned}$$

where r_{ij} is the incremental return obtained when the investment in project i is increased by k_{ij}. If the incremental return decreases as the level of investment increases, the utility function will be concave. In that case, we will have the following relationship:

$$r_{ij} \ge r_{ij+1} \text{ or } r_{ij} - r_{ij+1} \ge 0.$$

Thus,

$$Y_{ij} \ge Y_{ij+1} \text{ or } Y_{ij} - Y_{ij+1} \ge 0.$$

So that only the first n increments ($j = 1, 2, \ldots, n$) that produce the highest returns are used for project i. Figure 4 shows an example of a concave investment utility function.

If the incremental returns do not define a concave function, $f(x_i)$, then one has to introduce the inequality constraints presented above into the optimisation model. Otherwise, the inequality constraints may be left out of the model, since the first inequality, $Y_{ij} \ge Y_{ij+1}$, is always implicitly satisfied for concave functions. Our objective is to maximise the total return. That is,

$$\text{Maximise } Z = \sum_i \sum_j r_{ij} Y_{ij}$$

Subject to the following constraints:

$$x_i = b_i + \sum_j k_{ij} Y_{ij} \quad \forall i$$

$$b_i \le x_i \le M_i \quad \forall i$$

$$Y_{ij} \ge Y_{ij+1} \quad \forall i, j$$

$$\sum_i x_i \le B$$

$$x_i \ge 0 \quad \forall i$$

$$Y_{ij} = 0 \text{ or } 1 \quad \forall i, j$$

Now, suppose we are given four projects (i.e., $N = 4$) and a quality budget limit of \$10 million. The respective investments and returns are shown in Table 1, Table 2, Table 3, and Table 4.

Figure 4 Utility curve for investment yield

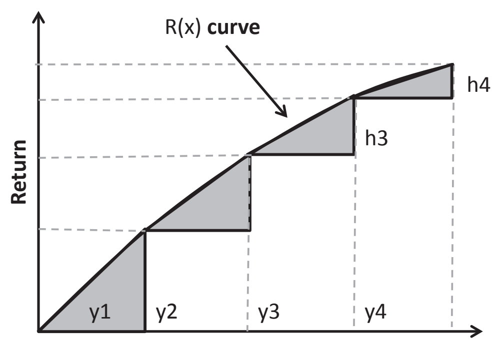


Table 1 Investment data for Project 1 for capital rationing

Stage (j)	y_{ij} *Incremental investment*	x_1 *Level of investment*	r_{ij} *Incremental return*	$R(x_1)$ *Total return*
0	-	0	-	0
1	0.80	0.80	1.40	1.40
2	0.20	1.00	0.20	1.60
3	0.20	1.20	0.30	1.90
4	0.20	1.40	0.10	2.00
5	0.20	1.60	0.10	2.10

Table 2 Investment data for Project 2 for capital rationing

Stage (j)	y_{2j} *Incremental investment*	x_2 *Level of investment*	R_{2j} *Incremental return*	$R(x_2)$ *Total return*
0	-	0	-	0
1	3.20	3.20	6.00	6.00
2	0.20	3.40	0.30	6.30
3	0.20	3.60	0.30	6.60
4	0.20	3.80	0.20	6.80
5	0.20	4.00	0.10	6.90
6	0.20	4.20	0.05	6.95
7	0.20	4.40	0.05	7.00

Table 3 Investment data for Project 3 for capital rationing

Stage (j)	y_{3j} *Incremental investment*	x_3 *Level of investment*	r_{3j} *Incremental return*	$R(x_3)$ *Total return*
0	0	-	-	0
1	2.00	2.00	4.90	4.90
2	0.20	2.20	0.30	5.20
3	0.20	2.40	0.40	5.60
4	0.20	2.60	0.30	5.90
5	0.20	2.80	0.20	6.10
6	0.20	3.00	0.10	6.20
7	0.20	3.20	0.10	6.30
8	0.20	3.40	0.10	6.40

Table 4 Investment data for Project 4 for capital rationing

Stage (j)	y_{4j} *Incremental investment*	x_4 *Level of investment*	r_{4j} *Incremental return*	$R(x_4)$ *Total return*
0	-	0	-	0
1	1.95	1.95	3.00	3.00
2	0.20	2.15	0.50	3.50
3	0.20	2.35	0.20	3.70
4	0.20	2.55	0.10	3.80
5	0.20	2.75	0.05	3.85
6	0.20	2.95	0.15	4.00
7	0.20	3.15	0.00	4.00

All of the values are in millions of dollars. For example, in Table 1, if an incremental investment of \$0.20 million from stage 2 to stage 3 is made in Project 1, the expected incremental return from the project will be \$0.30 million. Thus, a total investment of \$1.20 million in Project 1 will yield a total return of \$1.90 million. The question addressed by the optimisation model is to determine how many investment increments should be used for each project. That is, when should we stop increasing the investments in a given project? Obviously, for a single project, we would continue to invest as long as the incremental returns are larger than the incremental investments. However, for multiple projects, investment interactions complicate the decision so that investment in one project cannot be independent of the other projects. The LP model of the capital-rationing example was solved with LINDO software. The solution indicates the following values for Y_{ij}.

- Project 1:

  $$Y11 = 1, Y12 = 1, Y13 = 1, Y14 = 0, Y15 = 0$$

  Thus, the investment in Project 1 is $X1$ = \$1.20 million. The corresponding return is \$1.90 million.

- Project 2:

  $$Y21 = 1, Y22 = 1, Y23 = 1, Y24 = 1, Y25 = 0, Y26 = 0, Y27 = 0$$

  Thus, the investment in Project 2 is $X2$ = \$3.80 million. The corresponding return is \$6.80 million.

- Project 3:

  $$Y31 = 1, Y32 = 1, Y33 = 1, Y34 = 1, Y35 = 0, Y36 = 0, Y37 = 0$$

  Thus, the investment in Project 3 is $X3$ = \$2.60 million. The corresponding return is \$5.90 million.

- Project 4:

  $$Y41 = 1, Y42 = 1, Y43 = 1$$

  Thus, the investment in Project 4 is $X4$ = \$2.35 million. The corresponding return is \$3.70 million.

The total investment in all four projects is \$9,950,000. Thus, the optimal solution indicates that not all of the \$10,000,000 available should be invested. The expected return from the total investment is \$18,300,000. This translates into an 83.92% return on investment.

The optimal solution indicates an unusually large return on total investment. In a practical setting, expectations may need to be scaled down to fit the realities of the project environment. Not all optimisation results will be directly applicable to real situations. Possible extensions of the above model of capital rationing include the incorporation of risk and time value of money into the solution procedure. Risk analysis would be relevant, particularly for cases where the levels of returns for the various levels of investment are not known with certainty. The incorporation of time value of money would be useful if the investment analysis is to be performed for a given planning horizon. For example, we might need to make investment decisions to cover the next five years rather than just the current time.

6 Conclusions

As mentioned at the beginning of this paper, budgeting and capital rationing processes convey important messages about the investment potential of a project. The techniques presented here offer additional tools for assessing how and where limited resources should be directed. The outputs of the computational analysis can serve as a plan for resource expenditure on quality pursuit, a quality targeting criterion, a projection of quality policy, a basis for quality control, a performance measure, a standardisation of quality goal, and an incentive for improvement of organisational practices.

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