HOPS Short Course: Traveling Water Waves

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Abstract In this contribution we discuss Higher Order Perturbation of Surfaces (HOPS) methods with particular application to traveling water waves. The Transformed Field Expansion method (TFE) is discussed as a method for handling the unknown fluid domain. The procedures for computing Stokes waves and Wilton Ripples are compared. The Lyapunov-Schmidt procedure for the Wilton Ripple is presented explicitly in a simple, weakly nonlinear model equation.

1 Introduction

Traveling water waves have been studied for over a century, most famously by Stokes, for whom weakly-nonlinear periodic waves are now named [1, 2, 3]. In his 1847 paper, Stokes expanded the wave profile as a power series in a small parameter, the wave slope, a technique which has since become commonplace. This classic perturbation expansion, which we will refer to as the Stokes' expansion, has been applied to the water wave problem numerous times [4, 5, 6, 7, 8, 9]. When the effect of surface tension is included, the expansion may be singular. This singularity, due to a resonance between a long and a short wave, was noted first by Wilton [10] and has been studied more recently in [11, 12, 13, 14, 15].

In these lecture notes, we explain how traveling water waves may be computed using a High Order Perturbation of Surfaces (HOPS) approach, which numerically computes the coefficients in an amplitude-based series expansion of the free surface. For the water wave problem, a crucial aspect of any numerical approach is the method used to handle the unknown fluid domain. Popular examples include Boundary Integral Methods [16, 17], conformal mappings [18, 19], and series computations of the Dirichelet-to-Neumann operator [20, 21]. Here we discuss an al-

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ternative approach, in which the solution is expanded using the Transformed Field Expansion (TFE) method, developed in [22, 23].

The TFE method has been used to compute traveling waves on both two-dimensional (one horizontal and one vertical dimension) and three-dimensional fluids, both for planar and short-crested waves [23]. Short-crested wave solutions to the potential flow equations have been computed without surface tension [22, 24, 25] and with surface tension [26]. They have also been studied experimentally [27, 28].

The TFE method computes a Stokes expansion of the water wave to all orders. There is a long history of numerical implementation of the Stokes expansion to simulate water waves [29], with the approaches of [30, 31] and [22] of greatest relevance in the present context. However, only the TFE methods can be *rigorously* shown to converge (proof in [32]; see Figure 1 in [22] for an explicit demonstration of the ill-conditioning present in the algorithm of [30]), and thus be completely reliable for numerical simulation. In a recent paper, Wilkening and Vasan discuss this type of numerical ill-conditioning, which is due to floating-point cancellation, and show how the use of multi-precision arithmetic allows one to use the Craig-Sulem expansion to compute traveling waves (this expansion would otherwise be victim to the same ill-conditioning of Roberts et. al.) [33].

The implicit assumption of the Stokes' expansion, that solutions are analytic in the wave slope parameter ε, can also be exploited to derive weakly nonlinear model equations. In the case of shallow water waves with surface tension, well-known weakly-nonlinear models include the 5th order KdV equation [34] and the KP equation [35]. Without surface tension, Boussinesq type models have been used to study weakly nonlinear short-crested waves, for example in [36]. Recently, analogues of both the 5th order KdV [37] and KP equations [38] have been derived for deep water gravity-capillary waves. In this work a weakly nonlinear model will be used to present the details required to compute a high order Stokes expansion, as in [15]. This model allows one to avoid some of the technical details of the full Euler equations, but still captures the essence of the methods required to compute Stokes waves and Wilton ripples.

In these notes, we study High Order Perturbation of Surfaces (HOPS) methods for computing water waves. We begin by discussing methods for dealing with the unknown fluid domain in the water wave problem - Taylor Series, Operator Expansions, and the Transformed Field Expansion. Next, we discuss the Lyapanov-Schmidt reduction used to compute the series to all orders - first for Stokes waves in the TFE expansion, second for Wilton ripples in a weakly nonlinear model.

2 Water Waves

The widely accepted model for the motion of waves on the surface of a large body of water with constant surface tension, and in the absence viscosity, are the Euler equations

$$\phi_{xx} + \phi_{zz} = 0, \qquad z < \varepsilon\eta, \tag{1a}$$
$$\phi_z = 0, \qquad z = -H, \tag{1b}$$
$$\eta_t + \varepsilon\eta_x\phi_x = \phi_z, \qquad z = \varepsilon\eta, \tag{1c}$$
$$\phi_t + \frac{\varepsilon}{2}\left(\phi_x^2 + \phi_z^2\right) + \eta - \sigma\left(\frac{\eta_{xx}}{(1+\varepsilon^2\eta_x^2)^{3/2}}\right) = 0, \qquad z = \varepsilon\eta, \tag{1d}$$

where η is the free–surface displacement and ϕ is the velocity potential. These equations describe the motion of an inviscid incompressible fluid undergoing an irrotational motion. System (1) has been nondimensionalized as in [23, 39]. For HOPS methods, the wave slope $\varepsilon = A/L$ is assumed to be small (A is a typical amplitude and L, the characteristic horizontal length, is chosen in the non–dimensionalization so that the waves have spatial period 2π). Also, the vertical dimension has been non-dimensionalized using the wavelength, so the quantity H is non-dimensional ($H = h/L$). For simplicity's sake, in these notes we will consider the deep water limit $H \to \infty$. To compute traveling waves, the time dependence of (1) is prescribed, using the ansatz

$$\eta(x,t) = \eta(x+ct) \qquad \text{and} \qquad \phi(x,z,t) = \phi(x+ct,z).$$

To proceed further, one must choose a method to handle the fundamental difficulty of the water wave problem: that the problem domain is unknown.

3 The Fluid Domain

In this section, approaches for dealing with the vertical dependence of the water wave problem are discussed. Of course there are many other methods (for example conformal mapping or boundary integral methods). Here the aim is to discuss boundary perturbation methods and so we will restrict our attention to three methods based on series expansions: Taylor series, Operator Expansions, and the Transformed Field Expansion method.

3.1 Taylor Series

The most classic approach for the fluid domain is the one used originally by Stokes, and later by Wilton, and much later numerically by for example Roberts et. al. [1, 10, 30, 24]. The idea is simple, presume that the potential is analytic in the vertical dimension near $z = 0$, and Taylor expand the potential, and thus the boundary conditions, about this flat boundary. The resulting problem will have an infinite degree nonlinearity, but be posed on a half plane (or strip in finite depth). The new boundary conditions are

$$c\eta_x - \phi_z + \varepsilon\partial_x\left(\sum_{n=0}^{\infty}\frac{\varepsilon^n}{(n+1)!}\eta^{n+1}\partial_z^n\partial_x\phi\right) = 0, \qquad \text{at} \qquad z=0, \quad (2a)$$

$$c\phi_x + \eta - \sigma\partial_x\left(\frac{\partial_x\eta}{(1+\varepsilon|\partial_x\eta|^2)^{1/2}}\right) + \sum_{n=1}^{\infty}\frac{\varepsilon^n}{n!}\eta^n\partial_z^n\phi_t + \ldots$$
$$+\frac{\varepsilon}{2}\left(\sum_{n=0}^{\infty}\frac{\varepsilon^n}{n!}\eta^n\partial_z^n\partial_x\phi\right)^2 + \frac{\varepsilon}{2}\left(\sum_{n=0}^{\infty}\frac{\varepsilon^n}{n!}\eta^n\partial_z^n\phi_z\right)^2 = 0, \qquad \text{at} \qquad z=0. \quad (2b)$$

Laplace's equation is exactly solvable for the potential in the lower half plane, given its trace at $z=0$, denoted here as $\Phi(x)=\phi(x,0)$:

$$\phi(x,z) = \mathscr{F}^{-1}\left\{\mathscr{F}\{\Phi(x)\}\, e^{|k|z}\right\}.$$

Thus we can write equations (2) in terms of Φ, and replacing $\phi_z(x,0) = \mathscr{T}\Phi(x)$, effectively eliminating the vertical dependence. The operator $\mathscr{T}$ is then defined by setting $z=0$ above, or in terms of the Fourier transformed variables,

$$\hat{\phi}(k,0) = |k|\hat{\Phi}(k) = \widehat{\mathscr{T}\Phi}(k).$$

Unfortunately, this type of expansion, although effective for weakly nonlinear models (where the series are truncated at small order), are ill-conditioned when large numbers of terms are kept in the nonlinearity. Many terms in the series almost cancel, meaning that precision is lost in the result of combining these sums. To fix such numerical instability, in fixed precision storage types, one desires to avoid large-degree nonlinearities, which can be done using the TFE approach.

3.2 Operator Expansions

A popular alternative to the Taylor series approach of the previous section is to map the problem to the free surface, via the Dirichlet-to-Neumann operator (DNO). For example, one can write the water wave problem using Zakharov's canonical variables [40] $\eta(x,t)$ and

$$\xi(x,t) := \phi(x,\eta(x,t),t)$$

(the displacement and the *surface* velocity potential). This formulation was made explicit by Craig & Sulem [41], with the introduction of the Dirichlet–Neumann operator (DNO),

$$G(\eta)\xi := (\partial_z\phi - (\partial_x\eta)\partial_x\phi)_{z=\eta},$$

which maps Dirichlet data, ξ, to Neumann data at the interface η. In terms of this operator the evolution equations (1) can be *equivalently* stated as

$$\partial_t \eta = G(\eta)\xi, \tag{3a}$$
$$\partial_t \xi = -\eta + \sigma\eta_{xx} - A(\eta)B(\eta,\xi), \tag{3b}$$

where

$$A(\eta) = \frac{1}{2(1+(\partial_x\eta)^2)} \tag{3c}$$
$$B(\eta,\xi) = (\partial_x\xi)^2 - (G(\eta)\xi)^2 - 2(\partial_x\eta)(\partial_x\xi)G(\eta)\xi + \sigma\partial_x\left(\frac{\partial_x\eta}{(1+\varepsilon^2(\partial_x\eta)^2)^{1/2}} - \partial_x\eta\right). \tag{3d}$$

An amplitude expansion in this setting asks one to expand the operator $G(\eta)$, and thus it is natural to refer to such a HOPS method as an operator expansion; it fundamentally relies on expansion of the DNO [42]. We refer the interested reader to [42] for the full details, including the fundamental difficulty of computing the first variation of the DNO (see also [43, 44]). Such methods can be numerically implemented, provided that one is very careful with floating-point cancellation, see [33] for discussion of a multi-precision implementation using the operator expansion of Craig & Sulem.

3.3 Transformed Field Expansion

The TFE method begins with a very similar idea to the Taylor series approach, solving for the potential below a fixed depth. The change is that instead of the depth being $z = 0$, we choose some depth below the minimum of the fluid interface, $z = -a$. As before, Laplace's equation can be solved below this interface, yielding a new boundary condition at $z = -a$ instead of $z = -\infty$:

$$\phi_z = \mathscr{T}\phi,$$

with $\mathscr{T}$ defined as in the previous section, simply evaluating the solution of Laplace's equation at $z = -a$ instead of $z = 0$. Although the domain for Laplace's equation is still unknown, it is now bounded and small ($-a < z < \varepsilon\eta$). The next step is to change variables, mapping the domain to a strip. Rather than conformal mapping, which would leave Laplace's equation unchanged, the TFE approach uses the transformation

$$\tilde{z} = a\left(\frac{z-\varepsilon\eta}{a+\varepsilon\eta}\right),$$

which naturally generalizes to three dimensional fluids. In this setting, we can write Euler's equations for the transformed field $u(\tilde{x},\tilde{z})$, using

$$u(\tilde{x},\tilde{z}) = \phi\left(\tilde{x}, \left(\frac{a+\varepsilon\eta}{a}\right)\tilde{z} + \varepsilon\eta\right).$$

Because this transformed field u mixes x and z dependence, derivatives transform nonlinearly

$$u_{\tilde{z}} = \phi_z \left(\frac{a+\varepsilon\eta}{a} \right) \qquad u_{\tilde{x}} = \phi_x + \left(\left(\frac{a+\varepsilon\eta_x}{a} \right) \tilde{z} + \varepsilon\eta_x \right) \phi_z$$

This transformation increases the degree of the nonlinear terms in the boundary conditions, but only to cubics. The cost of this lower degree nonlinearity is that Laplace's equation becomes inhomogeneous, and the new system is of the form

$$u_{xx} + u_{zz} = F, \qquad -a < z < 0, \tag{4a}$$
$$u_z - \mathscr{T} u = J, \qquad z = -a, \tag{4b}$$
$$c\eta_x - \phi_z = Q, \qquad z = 0, \tag{4c}$$
$$c\phi_x + \eta - \sigma\eta_{xx} = R, \qquad z = 0, \tag{4d}$$

where all the nonlinear terms are included in F, J, Q, and R, whose degree is at most cubic, and are reported in [22, 23]. Solving such an inhomogeneous equation in the vertical direction is more difficult than a homogeneous one and is typically done numerically. However the result can be shown to be numerically stable for boundary perturbation methods, due to the finite-degree nonlinearity (a stark contrast from the previous section) [32]. It is also not so expensive, since a can be chosen to be small, thus one needs only a few points when solving for the the vertical dependence.

4 Boundary Perturbation

In the following sections we describe how one might compute traveling solutions to (4), using a perturbation expansion, about the flat state. The overall method will be described for the water wave problem, in which we will discuss the computation of Stokes waves. The extra Lyapunov-Schmidt reduction required to compute Wilton Ripples will be presented in the case of a simpler, weakly nonlinear family of model equations, as in [15].

4.1 Stokes Waves

The boundary perturbation method discussed here is based on an amplitude expansion about the linear solution of the water wave problem:

$$u_{xx} + u_{zz} = 0, \qquad -a < z < 0, \tag{5a}$$
$$u_z - \mathscr{T} u = 0 \qquad z = -a, \tag{5b}$$
$$c\eta_x - \phi_z = 0, \qquad z = 0, \tag{5c}$$
$$c\phi_x + \eta - \sigma\eta_{xx} = 0, \qquad z = 0. \tag{5d}$$

The linear problem (5), can be solved exactly, and has dispersion relation

$$\omega(k)^2 = c^2k^2 = |k|(1+\sigma k^2).$$

Thus for each wavenumber $k \in \mathbb{R}$ there are two speeds $c = \pm c_p(k)$, where $c_p(k) = \omega(k)/k$ is the phase speed. Similarly at each speed, there are two possible wave numbers, see figure 1. If we restrict to a periodic domain, $k \in \mathbb{Z}$, most of the time there is only one wavenumber which travels at a given speed, or equivalently the two wave numbers which travel at a given speed do not share a period. When only one wavenumber both fits in a prescribed period and travels at a given speed, the leading order solution is

$$\eta_0(x) = e^{ix} + *, \tag{6a}$$
$$u_0(x,0) = ic_0e^{ix} + *, \tag{6b}$$
$$c_0 = \sqrt{(1+\sigma)}, \tag{6c}$$

in which $*$ corresponds to the complex conjugate of the preceding terms. We call the nonlinear solution for which (6) is the leading order term a Stokes' wave. The goal of boundary perturbation is now to construct a series

$$u = \sum_{n=1}^{\infty} \varepsilon^n u_n, \qquad \eta = \sum_{n=1}^{\infty} \varepsilon^n \eta_n, \qquad \text{and} \qquad c = \sum_{n=0}^{\infty} \varepsilon^n c_n,$$

with u_0, η_0, c_0 given by (6). To compute the terms in this series η_j, u_j and c_j, which we call corrections, we first substitute the above ansatz into (4), and then collect consecutive powers of ε. At each perturbation order, one must then solve

$$\Delta u_n = F_n, \qquad -a < z < 0, \tag{7a}$$
$$\partial_z u_n - T u_n = J_n, \qquad z = -a, \tag{7b}$$
$$c_0\partial_x\eta_n - \partial_z u_n = Q_n - c_n\partial_x\eta_0, \qquad z = 0, \tag{7c}$$
$$c_0\partial_x u_n + (1-\sigma\Delta_x)\eta_n = R_n - c_n\partial_x u_0, \qquad z = 0, \tag{7d}$$

where the formula for F_n, J_n, Q_n, and R_n appear in [22, 23], and are known functions of η_j, u_j, c_j for $j < n$. To compute u_n, η_n from (7), one must first ensure that the right hand side is in the range of the linear operator (on the left). The linear problem has non-trivial solution, thus one must impose a solvability condition for each independent solution of the adjoint problem, which we call ψ. Stokes waves correspond to the case where the adjoint problem has a one-dimensional solution space.

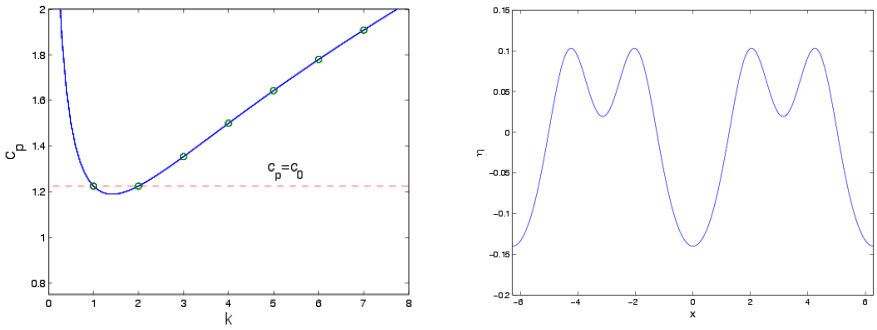

Fig. 1 LEFT: The phase speed c_p for $\sigma = 1/2$, for which there is a Wilton ripple, between $k = 1$ and $k = 2$. The wave at $k = 1$ travels at the same speed as one of its harmonics (at $k = m$) only if $\sigma = 1/m$. For all other Bond numbers the solution is a Stokes' wave. RIGHT: An example of two periods a ripple for $\sigma = 1/2$, computed in equation (10).

The Lyapunov-Schmidt reduction for Stokes waves is quite simple. Defining $v_n = (0,0,u_n,\eta_n)^T$ and $G_n = (F_n,J_n,Q_n,R_n)^T$, the Fredholm alternative theorem says that there are solutions of (7) if and only if

$$\langle \psi, G_n - c_n \partial_x v_0 \rangle = 0. \tag{8}$$

This equation is linear in c_n. After calculating c_n from (8), one can then invert the linear operator upon its range, calculating u_n and η_n, and proceed to the next order. This procedure is done numerically in [22, 23, 32].

4.2 Wilton Ripples

When the adjoint problem has more than one independent solution, the Lyapunov-Schmidt reduction required to compute solutions is more complicated. For deep water waves on a two-dimensional fluid this occurs when the Bond number $\sigma = \frac{1}{m}$, with $m \in \mathbb{N}$. For this countable set of Bond numbers, there are two nontrivial solutions to the linear problem, which one might express as

$$\eta_0 = e^{ix} + \beta_0 e^{imx} + *, \qquad \text{and} \qquad u_0(x,0) = ic_0 e^{ix} + \beta_0 ic_0 e^{imx} + *, \tag{9}$$

both of which move at speed $c_0 = \sqrt{1+\sigma}$. From a linear perspective, arbitrary values of β_0 are allowed. Only particular values of β_0 correspond to nonlinear solutions however. Rather than present the perturbation expansion for the full water wave problem, where the bookkeeping becomes quite arduous, we will present the expansion in a simpler model,

$$c\eta_x - \mathscr{L}\eta_x + \varepsilon(\eta^2)_x = 0, \tag{10}$$

where $\mathscr{L}$ is a pseudo-differential operator, whose Fourier symbol is

$$\hat{\mathscr{L}}(k) = c_p(k) = \sqrt{\frac{1}{|k|} + \sigma|k|},$$

to mimic the water wave problem (we are presenting here the results of [15], for a particular choice of $\mathscr{L}$). The leading order solution will be identical to the η portion of (9). A nonlinear solution is then sought in the form

$$\eta = \sum_{n=0}^{\infty} \varepsilon^n \eta_n, \qquad \text{and} \qquad c = \sum_{n=0}^{\infty} \varepsilon^n c_n.$$

This ansatz results in a series of linear equations for η_n,

$$(c_0 - \mathscr{L})\partial_x \eta_n = -\partial_x \left(\sum_{j=0}^{n-1} \eta_j \eta_{n-1-j} \right) - \sum_{j=0}^{n-1} c_{n-j} \partial_x \eta_j. \tag{11}$$

Just as with Stokes waves, we seek to compute the corrections (η_j, c_j) at each order, by first enforcing that the right hand side of (11) is in the range of the linear operator (on the left). In the ripple case there are two solutions to the adjoint linear problem, and we will need to enforce two solvability conditions. The speed corrections c_n are still available for one unknown at each order, but since there are now two conditions, we need a second unknown. The second unknown is a coefficient of the homogeneous solutions to (11). We choose to add these in the following way:

$$\eta_n = \eta_n^{\perp} + \beta_n e^{imx},$$

where $\eta_n^{\perp}$ is not supported at $k = 1$ or $k = m$, and is a particular solution of (11). The total correction η_n is a sum of this particular solution and a homogeneous contribution at frequency $k = m$.

The n$^{\text{th}}$ correction includes no contribution from the other homogeneous solution (at $k = 1$). This choice defines the total of the Fourier transform of the solution at $k = 1$ to be 1, and thus $\sum_{j=0}^{\infty} \varepsilon^j \beta_j$ is defined to be the Fourier transform of the solution at $k = m$. With these definitions, the solution is unique.

The series can be computed for all m, but for simplicity of presentation, we will only present the case considered by Wilton himself, $m = 2$. Larger values of m require similar manipulations; see a complete treatment of $m = 3$ in [15]. For $m = 2$, the character of the entire perturbation series can be determined by considering only the first three orders. The $O(\varepsilon^0)$ solution is identical to the leading order free surface of the Euler equations, here in (9), as is the leading order speed c_0. At the next order, one must impose that equation (11) is solvable, that the right hand side of (11) is orthogonal to both $\psi_j = e^{ik_j x}$:

$$\left\langle e^{ix}, (\eta_0^2)_x + c_1 \partial_x \eta_0 \right\rangle = 0,$$
$$\left\langle e^{2ix}, (\eta_0^2)_x + c_1 \partial_x \eta_0 \right\rangle = 0.$$

This can be written simply in terms of β_0 and c_1:

$$c_1 + 2\beta_0 = 0,$$
$$c_1\beta_0 + 1 = 0.$$

Thus $c_1 = \pm\sqrt{2}$ and $\beta_0 = \mp\sqrt{2}/2$. These numbers depend only on the nonlinearity, not the linear operator; they will be different for the full water wave problem. At later orders, the solvability conditions are

$$\left\langle e^{ix}, 2(\eta_0\eta_{n-1})_x + c_n\partial_x\eta_0 + c_1\partial_x\eta_{n-1} + \partial_x\left(\sum_{j=1}^{n-2}\eta_j\eta_{n-1-j}\right) + \sum_{j=1}^{n-2}c_{n-j}\partial_x\eta_j \right\rangle = 0,$$
$$\left\langle e^{2ix}, 2(\eta_0\eta_{n-1})_x + c_n\partial_x\eta_0 + c_1\partial_x\eta_{n-1} + \partial_x\left(\sum_{j=1}^{n-2}\eta_j\eta_{n-1-j}\right) + \sum_{j=1}^{n-2}c_{n-j}\partial_x\eta_j \right\rangle = 0.$$

In the above solvability conditions, the unknowns c_n and β_{n-1} appear linearly, and only in the first three terms in the second argument of the inner product. The resulting solvability conditions can be written as

$$\begin{pmatrix} 1 & 2 \\ \beta_0 & c_1 \end{pmatrix}\begin{pmatrix} c_n \\ \beta_{n-1} \end{pmatrix} = \mathbf{D}_n, \tag{12}$$

where $\mathbf{D}_n$ are known functions of the previous corrections.

$$\mathbf{D}_n = \begin{pmatrix} \left\langle e^{ix}, \partial_x\left(\sum_{j=1}^{n-2}\eta_j\eta_{n-1-j}\right) + \sum_{j=1}^{n-2}c_{n-j}\partial_x\eta_j \right\rangle \\ \left\langle e^{2ix}, \partial_x\left(\sum_{j=1}^{n-2}\eta_j\eta_{n-1-j}\right) + \sum_{j=1}^{n-2}c_{n-j}\partial_x\eta_j \right\rangle \end{pmatrix}$$

Since the matrix on the left-hand side of (12) is invertible, this equation is solvable at all orders, and we can formally construct the solutions. To use such an approach, of course one must also consider convergence of the series, see [15] for discussion of the convergence of the above series. An example of a Wilton ripple solution for $m = 1$ is in the right panel of Figure 1.

Wilton ripples can be thought of a special case of a resonant interaction, where both the spatial and temporal frequencies of a set of waves sum to zero:

$$k_1 + k_2 + \cdots + k_m = 0, \qquad \omega_1 + \omega_2 + \cdots + \omega_m = 0.$$

Such nonlinear interactions are commonly referred to as triads when they contain three waves, quartets with four waves, etc. [45, 46]. Wilton ripples have wave numbers and dispersion relations satisfying such a resonant interaction, where

$$k_1 = k_2 = \cdots = k_{m-1} \qquad \text{and} \qquad k_m = mk_1,$$

along with

$$\omega_1 = \omega_2 = \cdots = \omega_{m-1} \qquad \text{and} \qquad \omega_m = m\omega_1.$$

Although these notes present only the triad ripple, $m = 1$, where the solvability conditions are quadratic, the other resonances are similar, with the exception that the solvability condition include a cubic nonlinearity at $O(\varepsilon^2)$.

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