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Reliability Test Plans for Binary Responses - Excel Tool Guide

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The goal of the STAT COE is to assist in developing rigorous, defensible test strategies to more effectively quantify and characterize system performance and provide information that reduces risk. This and other COE products are available at www.AFIT.edu/STAT.

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Revision 1, 9 Oct 2018, Formatting and minor typographical/grammatical edits.

Executive Summary

This paper describes how to use the STAT COE tool in excel for planning and evaluating reliability tests when the response is binary (pass/fail). The methods used here utilize acceptance sampling plans commonly used in industry and are common in statistical software. This tool should be used in situations when practitioners do not have access to statistical software packages.

Keywords: operating characteristic curve, acceptance sampling, binary response, test plan comparison

Introduction

This paper describes how to use the STAT COE tool in excel for planning and evaluating reliability tests when the response is binary (pass/fail). This tool should be used in situations when practitioners do not have access to statistical software packages. The methods used here utilize acceptance sampling plans commonly used in industry and are common in statistical software. This tool should be used in place of the binomial nomograph. The appendix of this paper explains the calculations used to determine the test plans and to plot the resulting operating characteristic (OC) curves. Refer to Truett (2013) for questions on using OC curves and their interpretation. To generate a reliability test plan when the response is continuous, refer to Kensler (2014a, b). To compare test plans for continuous responses, refer to Harman (2015a, b).

The assumptions regarding the use of these sampling plans are as follows: 1) the response/outcome is binary (pass/fail, for example); 2) each run in the sample is independently executed and performed under the same test conditions; 3) the percent defective in the population is constant; and 4) the sample size n is fixed.

The goal of this paper is to describe how to use the tool, but first recall these important definitions:

- Acceptable quality limit (AQL): the largest percent considered acceptable (the level at which we would like to pass the system). This can be thought of as the objective for an acquisition system.
- Rejectable quality limit (RQL): the poorest level of quality that should be accepted (the level at which we would like to fail the system). This can be thought of as the threshold for an acquisition system.
- Producer’s risk: The probability of rejection/failing a good system (i.e. failing a system operating at or above the AQL).
- Consumer’s risk: The probability of acceptance/passing a bad system (i.e. accepting a system operating at or below the RQL).

This tool has two functions: generate a test plan given supplied inputs and compare test plans using OC curves. Each of these functions are described in the following two sections.

Get Test Plan

The first tab in the tool allows you to generate a test plan given inputs of the AQL, RQL, producer's risk, and consumer's risk. Ideally, a system operating at the AQL should have a high probability of acceptance and a system operating at the RQL should have a low probability of acceptance. The AQL and RQL should be entered as a percent so that $0 < RQL < AQL < 100$. For example, suppose we have a system with an objective reliability of 97% and threshold reliability of 95%. We set the producer's risk and consumer's risk at 0.05 and 0.10, respectively, meaning that there is a 5% chance of failing the test for a system that operates at or above the AQL and a 10% chance of passing the test for a system that operates at or below the RQL. These values are entered as shown in Figure 1.

Acceptable Quality Limit	97.00
Rejectable Quality Limit	95.00
Producer's Risk	0.05
Consumer's Risk	0.10

Get Test Plan

Figure 1. Data entry to generate test plan

By clicking the button "Get Test Plan," the resulting sampling plan, consisting of the sample size and acceptance number, is displayed (Figure 2).

Acceptance Sampling Plan	
Sample size, n	807
acceptance number, c	32

Figure 2. Test plan for supplied inputs

In addition, the resulting OC curve is displayed with the AQL and RQL overlaid on the same plot (Figure 3) as the green and red vertical lines, respectively. Note that the producer's and consumer's risks meet the specified requirements.

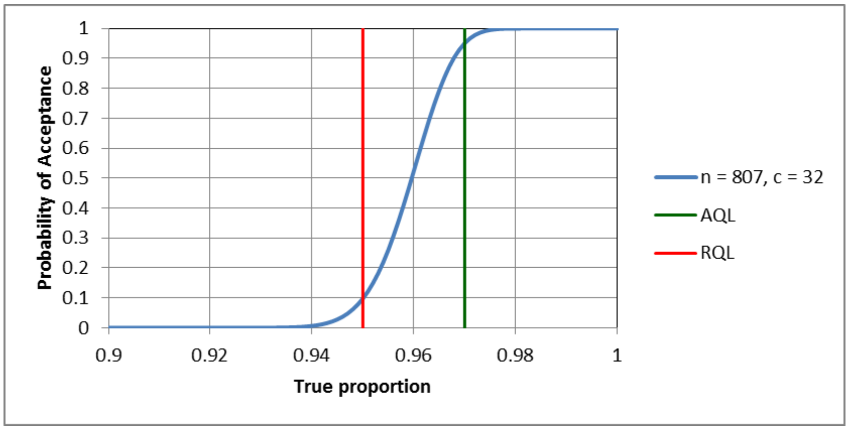

Figure 3. OC curve for resulting test plan

In addition to the OC curve, an interactive table is included in the tool that allows you to enter in a value of p, the quality level entered as a probability $(0 < p < 1)$, to determine the probability of acceptance (pass the test) or rejection (fail the test). This value p is entered into any of the orange cells as shown in Figure 4 and the resulting probabilities are shown in the blue cells. The first two rows of this table are automatically updated to include the AQL and RQL as entered in Figure 1.

p	Probability Pass Test	Probability Fail Test
0.9700	0.9514	0.0486
0.9500	0.0994	0.9006
0.9000	0.0000	1.0000
0.9750	0.9952	0.0048
0.9600	0.5267	0.4733

Figure 4. Interactive table to calculate probability pass or fail for supplied value of p

Compare Test Plans

The second tab in the tool allows you to compare up to three test plans by comparing their respective OC curves. To generate the OC curves for the test plans of interest, enter in the sample size and acceptance number in their respective boxes (Figure 5). You can enter in 3 or fewer sampling plans to plot their respective OC curves in the same plot for comparison. As an optional feature, you can enter in the AQL and/or RQL to include on the plot as well. The AQL and RQL should be entered as a percent. Consider the example previously where the objective and threshold reliability are 97% and 95%, respectively. In Figure 5, the AQL is entered as 97% and the RQL is entered as 95%. If both the AQL and RQL are entered, they should be entered such that:

$$0 < RQL < AQL < 100.$$

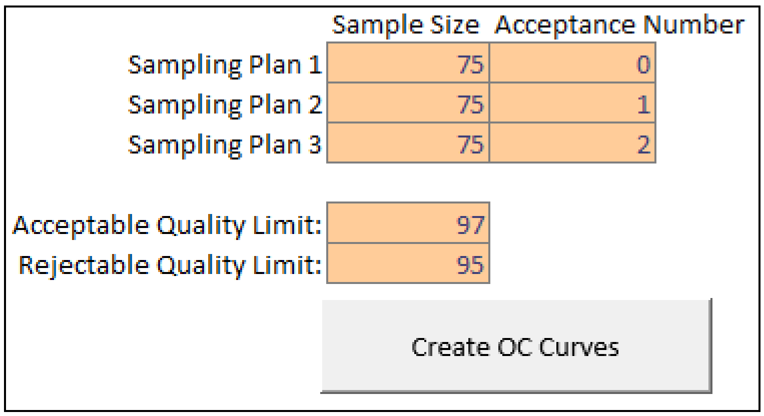

Figure 5. Data entry to compare test plans

Figure 6 shows the resulting OC curves for the plans entered in Figure 5, as well as the AQL and RQL. Note that these test plans do not perform well for the given AQL and RQL.

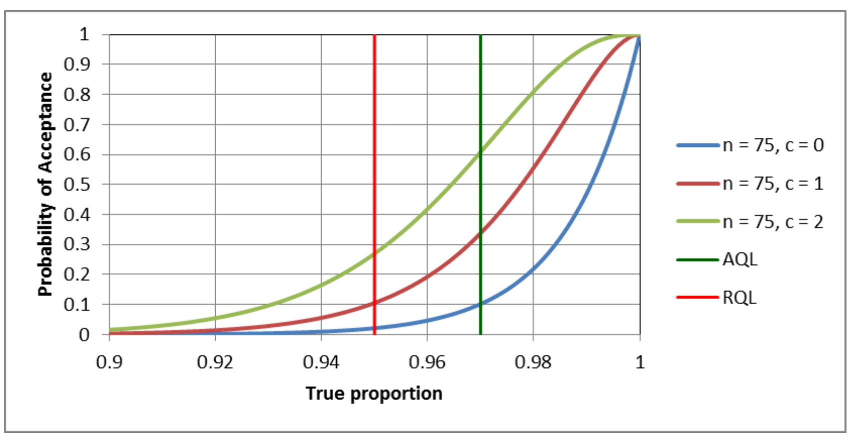

Figure 6. OC curves for three sampling plans

In addition to the OC curves, the tool also displays the probability of acceptance (pass test) and rejection (fail test) for the AQL and RQL values if they are entered (see Figure 7).

Sample Size	Acceptance Number	Quality Level	Probability Pass Test	Probability Fail Test
75	0	AQL	0.1018	0.8982
75	0	RQL	0.0213	0.9787
75	1	AQL	0.3380	0.6620
75	1	RQL	0.1056	0.8944
75	2	AQL	0.6083	0.3917
75	2	RQL	0.2697	0.7303

Figure 7. Summary results for AQL and RQL values

Conclusion

The excel tool presented in this paper is available on the STAT COE website at www.afit.edu/STAT or by email at COE@AFIT.edu. This tool can be used to determine test resources and to compare test plans. The appendix describes the calculations used to generate the test plan and to create the resulting OC curve.

References

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Appendix

The assumptions for a binomial random variable are: 1) a binary outcome; 2) a fixed sample size n; 3) independently selected runs from the population; and 4) constant probability of success p. If these assumptions hold for a test plan for a binary response, then the number of failures in a sample of size n has a binomial distribution. The probability of i failures is therefore defined as:

$$P(\#\, failures = i) = \binom{n}{i} p^i (1-p)^{n-i}$$

A system is accepted if the number of failures d is less than the acceptance number c. If the four assumptions described above hold, test plans for binary responses are determined by solving for the sample size n and acceptance number c through a system of two nonlinear equations:

$$1 - \alpha \geq \sum_{d=0}^{c} \binom{n}{d} p_1^d (1-p_1)^{n-d}$$

$$\beta \leq \sum_{d=0}^{c} \binom{n}{d} p_2^d (1-p_2)^{n-d},$$

where α is the probability of a type I error, β is the probability of a type II error, p_1 is the acceptable quality level (expressed as proportion defective), p_2 is the rejectable quality limit (expressed as proportion defective), and $\binom{n}{d} = \frac{n!}{(n-d)!d!}$. The probability of acceptance at the AQL should be at least as large as $1 - \alpha$. The probability of acceptance at the RQL should be no larger than β, the desired probability of acceptance at the RQL. We typically want this value to be small since $1 - \beta$ is the desired probability of rejection as the RQL. Because these equations are nonlinear and n and c must be integers, these values are determined through a search algorithm. The resulting test plan is the smallest sample size and acceptance number that satisfy these two equations.

The OC curve for a binary response utilizes the binomial distribution as well. The probability of acceptance (passing the test) is found by calculating the probability of observing c or fewer failures in the test across values of p from 0 to 1 where p represents the probability of failure:

$$P_a = P(d \leq c) = \sum_{d=0}^{c} \binom{n}{d} p^d (1-p)^{n-d}$$