Radar Detection Performance Using Design of Experiments – Case Study

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Bottom Line Up Front

- This case study is an application of experimental design to the test and evaluation of surface radars.
- It builds upon work done by the Naval Surface Warfare Center, Corona Division.
- We look back into a test that was considered a landmark in M&S-based acquisition and contrast the way one objective was evaluated to the way it could have been evaluated with experimental design.
- In the process, we explore the attributes of a well designed test and demonstrate the utility of experimental design for planning, designing, executing, and analyzing a test.
- *What can we learn from the data? What could we have done differently*? *What can we do different next time?*

An experimental design approach contributes to making the test more robust, efficient, and cost effective.

Trade-off Space

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Risk of accepting a "bad" system

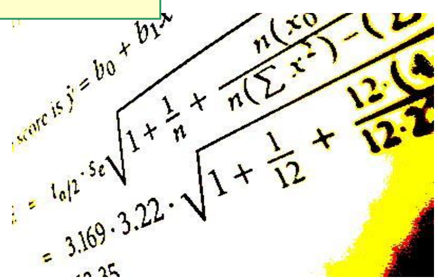
β

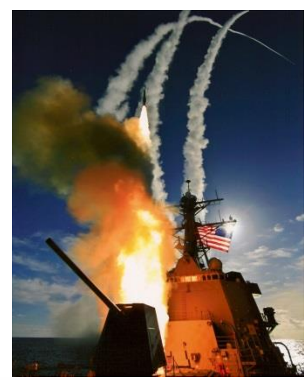
n

Adequate sample size

Effects on performance

δ / σ





α

Risk of rejecting a "good" system

The *Central Problem of Test* is to determine the true nature of the system, in all possible scenarios, with a finite number of samples that yield valid conclusions while minimizing the risk of error.

Test Background

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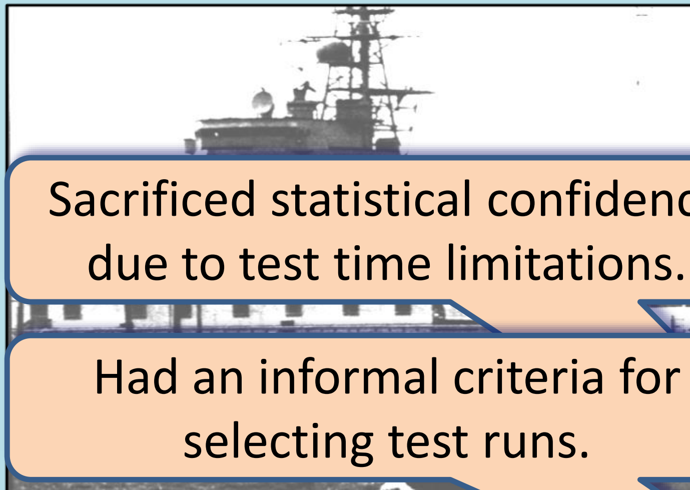
Test Operations



Sacrificed statistical confidence due to test time limitations.

Had an informal criteria for selecting test runs.

Analysis limited to pass/fail.

- Test events
  - 80 hrs total test time
  - 18 hrs of manned aircraft raids
  - 110 electronic attack (EA) techniques
  - 1900 simulated Anti Ship Cruise Missiles

One Objective

- Evaluate detection performance for a class of threat representative targets*
- Factors involved
  - Three target factors - A, B, F
  - Two environmental factors - C, D
  - One system factor - E
- Test strategy
  - 96 possible treatments
  - 30 samples per treatment required
  - 2880 total runs required
  - 96 hrs of test required-**not enough time!**
  - 670 runs conducted
- Assessment criteria - Pass/Fail

*Other objectives are beyond the scope of this brief; however, similar lessons apply.

Experimental Design Guidelines*

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Plan

1. Formulate a clear statement of the problem.
2. Identify the proper responses to be analyzed.
3. Identify the factors and their levels.

Design

4. Choose an appropriate experimental design.

Execute

5. Perform the test as outlined in the test matrix.

Analyze

6. Perform the appropriate statistical data analysis.
7. Reach valid and practical recommendations.

* Montgomery, D. C. (2013), *Design and Analysis of Experiments,8th ed.,* John Wiley & Sons.

Experimental Design Approach
Step 1 - Problem Definition

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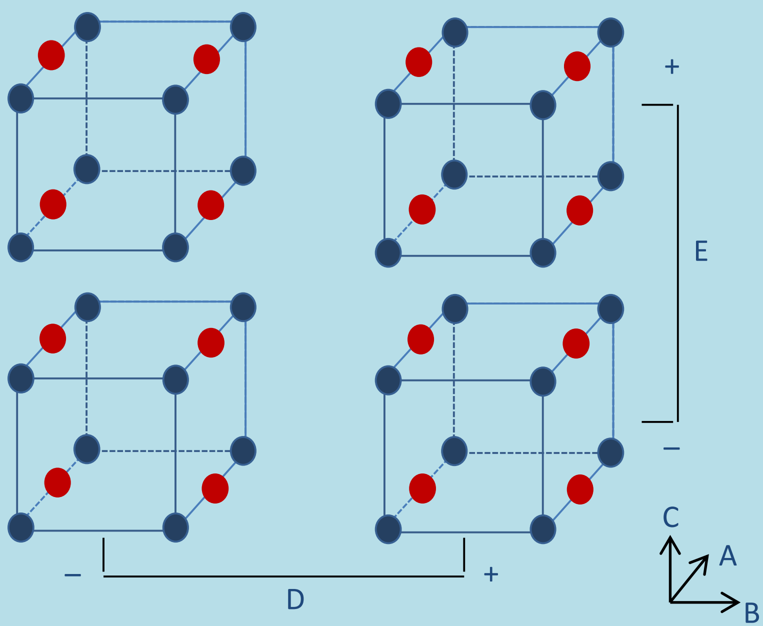
Performance Requirements



Statistical Parameters

Confidence level (α) - 0.05

Effect to detect (δ) – based on performance expectations

Variability (σ) - based on historical data

S/N (δ/σ) - 1.00 (for the case study)

We want to evaluate the effect of five factors* on detection performance.

*Six factors were of interest, but data for one factor was incomplete; therefore, the study was limited to five factors.

Experimental Design Approach

Step 1 - Historical Data Analysis

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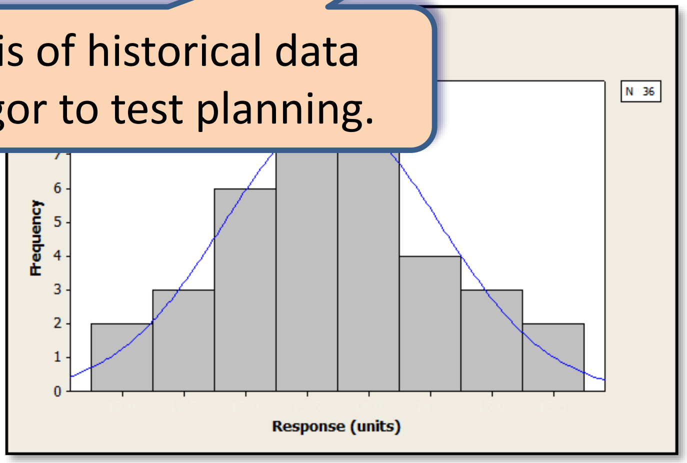
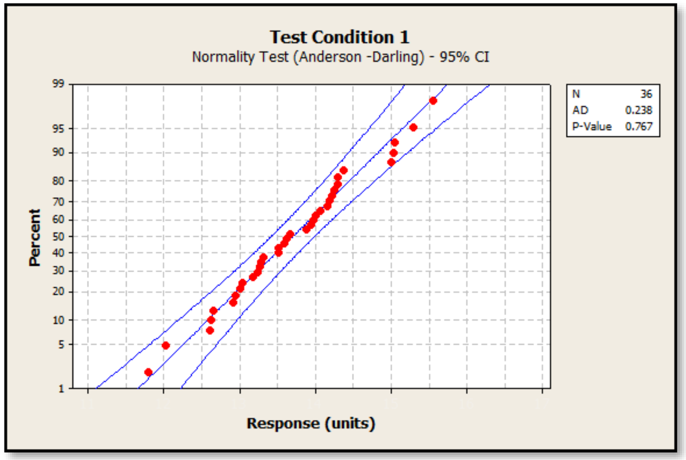
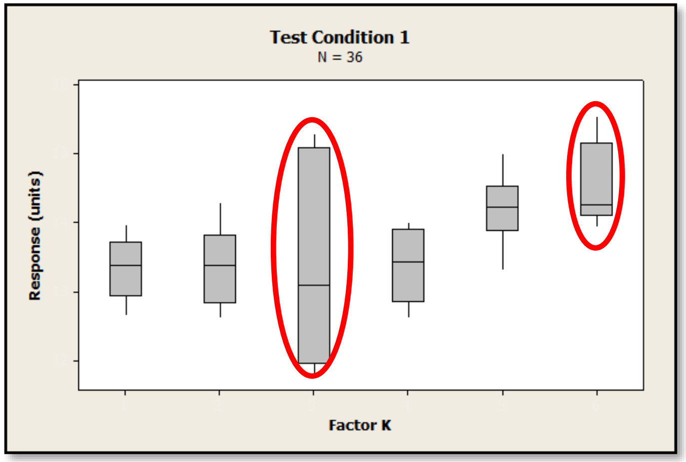
Analysis of historical data adds rigor to test planning.







Test Condition 1

Paired T-test for Difference of Means $(K_i - K_j)$

	1	2	3	4	5	6
1	-	X	X	X	X	X
2	-	-	X	X	X	X
3	-	-	-	X	X	X
4	-	-	-	-	X	X
5	-	-	-	-	-	X
6	-	-	-	-	-	-

Alpha = 0.05; x = p-value < 0.05

Experimental Design Approach
Steps 2 & 3 - Responses and Factors

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Continuous factors are preferred.

A single design can be used to evaluate multiple responses.

Covariates

Search Times, Power, Sensitivity, Material Readiness

actors

A (Continuous, 3-levels)

B (Categorical, 2-levels)

C (Categorical, 2-levels)

D* (Categorical, 2-levels)

E* (Categorical, 2-levels)

F^ (Fixed)

Responses

R1-Detection range

R2-Transition-to-track range

R3-Firm track range

R4-Engagement Range

$$e = f\left(\text{Factors}\right) + \varepsilon$$

Noise

Select appropriate factors.

Continuous responses are preferred.

* For large designs, these are hard(er)-to-change factors

^ Fixed during the original test due to test time limitations

Experimental Design Approach

Step 4 – Select a Design

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Some Experimental Design Alternatives*

Completely Randomized Designs; Model – ME + 2FI; Power (1 std. dev.) at α = 0.05

Design	Runs	Center Points	Power (%) (ME)	VIF	DOF			Std. Error (FDS=0.8)
					Model	LOF	PE	
MR-Res IV	12	0	27-28	1.1 - 7.0	10	1	0	1.5
MR-Res IV	92	5	39-99	1.0 – 1.5	15	12	64	0.7
2_V^{5-1}	16	0	-	1.0	15	0	0	-
2_V^{5-1}	96	5	50–98	1.0	15	16	64	0.6
D-Optimal	21	0	54–57	1.1	15	5	0	1.0
2^5	32	0	76	1.0	15	16	0	0.7
2^5	112	5	> 80	1.0	15	32	64	0.5
2 x 2^5	64	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2 x 2^5	144	5	98-99	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4 x 2^5	192	4	99	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Evaluate several designs and select one that has good properties and that is appropriate for the problem.

Legend:

ME – main effects
DOF – degrees-of-freedom
LOF – lack-of-fit
FDS – fraction of the design space

2FI – two factor interactions
VIF – variance inflation factor
PE – pure error

* Other designs were not explored due to data limitations.

Experimental Design Approach

Step 4 – Design Selection

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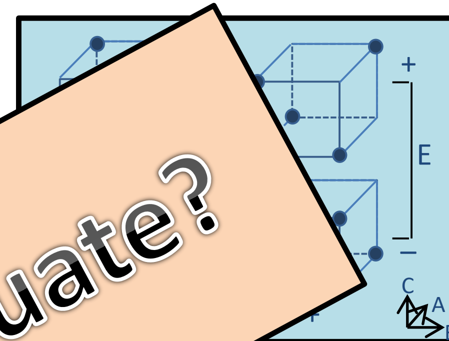
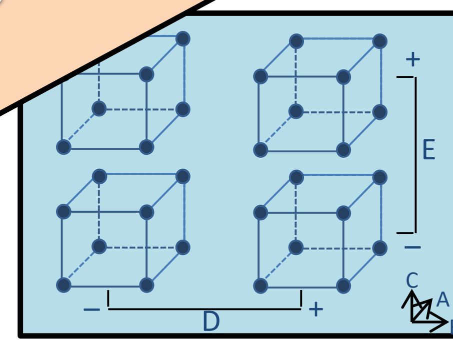
- Case I: 2_V^{5-1} fractional factorial
  - 16 runs
  - No degrees of freedom for estimating pure err
    lack-of-fit, or test of significance

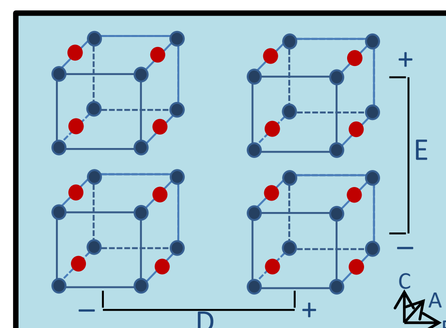


- Case II: 2^5 factori
  - 32 runs
  - No



Are these designs adequate?

- al + center point – 4 reps.
  - ter points allow testing for curvature and estimating pure error.



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Step 5 – Execution

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Run the test as specified in the test matrix.

Replicate, Randomize, and Block whenever possible.

…rix For Case III (2^5 factorial + c… point…

Run Order	Std. Order	Block	A	B	C	D	E	y
96	**1**	**Blk1**	**1**	**B2**	**C2**	**D1**	**E1**	**xxx**
35	2	Blk1	-1	B1	C1	D2	E1	xxx
101	3	Blk1	1	B1	C2	D2	E1	xxx
46	4	Blk1	1	B2	C1	D2	E1	xxx
32	5	Blk1	1	B2	C2	D1	E1	xxx
107	6	Blk1	-1	B2	C1	D2	E2	xxx
89	7	Blk1	0	B1	C2	D2	E1	xxx
…….	…….	…….	…….	…….	…….	…….	…….	…….
56	**n**	**Blk1**	**1**	**B2**	**C2**	**D1**	**E1**	**xxx**
…….	…….	…….	…….	…….	…….	…….	…….	…….
192	192	Blk2	-1	B1	C1	D1	E1	xxx

Experimental Design Approach

Step 6 - Statistical Analysis (Case I)

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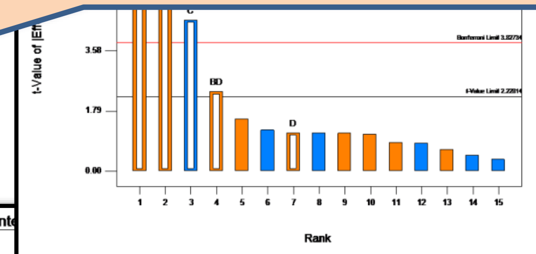
Factor D is not significant—a model is suspect.

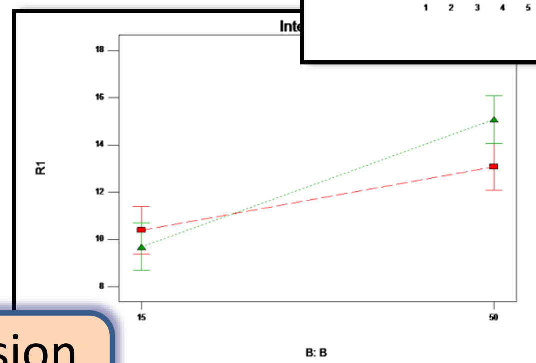
Experimental design affords studying interactions.

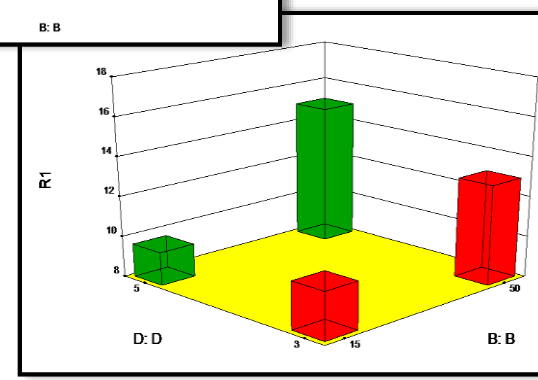
Analysis of Variance Table for Case I (2_V^{5-1} Fractional Factorial); p-value <0.1

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	167.26	5	33.45	25.70	< 0.0001
A-A	*66.59*	*1*	*66.59*	*51.15*	*< 0.0001*
B-B	*65.29*	*1*	*65.29*	*50.15*	*< 0.0001*
C-C	*26.37*	*1*	*26.37*	*20.26*	*0.0011*
D-D	*1.70*	*1*	*1.70*	*1.31*	*0.2794*
BD	*7.32*	*1*	*7.32*	*5.62*	*0.0392*
Residual	13.02	10	1.30		
Cor Total	180.28	15			

significan







Reduced Empirical Model (Coded Factors)

$$R1 = I + x_1A + x_2B - x_3C + x_4D + x_{24}BD$$

R^2 = ... 0.8917 Pred. R^2 = 0.8151 Adeq. Precision = 17.23

The empirical model is useful for tactical decision aids, training, and performance assessment.

Reference Mechanistic Model

$$R = \left(\frac{1}{(S/N)_t} \times \frac{P_t \times G \times \lambda^2}{(4\pi)^3 \times k} \times \frac{\sigma \times F^2}{L_t \times L_r \times L_{bs}{}^2 \times L_a{}^2 \times L_s{}^2} \times \frac{1}{T_s} \times \frac{\tau \times N}{L_p} \right)^{1/4}$$

Experimental Design Approach

Step 6 - Statistical Analysis (Case I)

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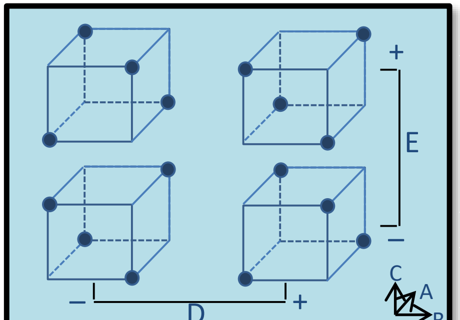
- Only factors A, B, and C and interaction BD are significant; factor E is dropped from consideration—*the sparsity of effects principle*.
- A Res V fractional factorial design contains a complete factorial in any subset of 4 factors—*the projection property*.
- We can combine the runs of fractional factorials to assemble a larger design (two blocks)—*sequential experimentation*

2_V^{5-1} Fractional Factorial



Dof for evaluation; model – ME + 2FI

Model	15
Residuals	0
Lack Of Fit	*0*
Pure Error	*0*
Total	15

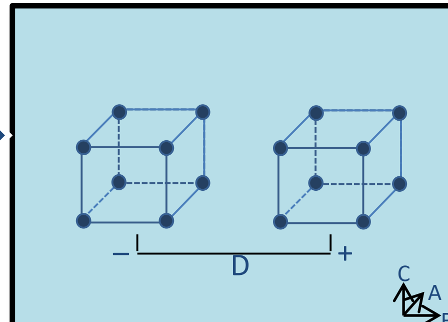
Unable to calculate power

Sparsity of effects

Projection property

Sequential experimentation

2^4 Factorial



Dof for evaluation; model – ME + 2FI

Model	10
Residuals	5
Lack Of Fit	*5*
Pure Error	*0*
Total	15

Power – 37% (1 std. dev.);
89% (2 std. dev.)

Experimental Design Approach
Step 6 - Statistical Analysis (Case II)

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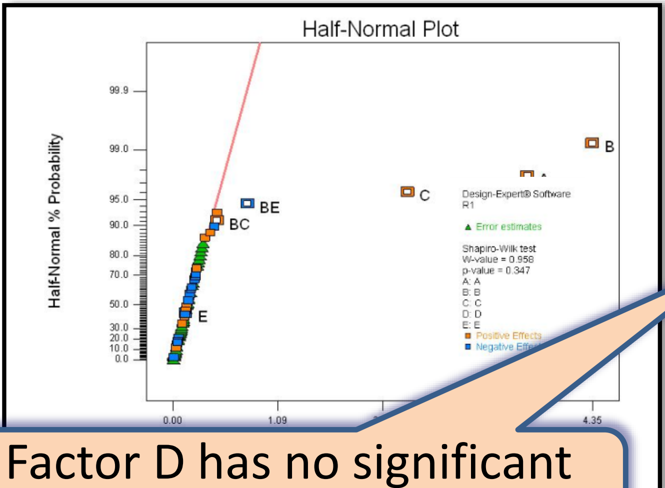
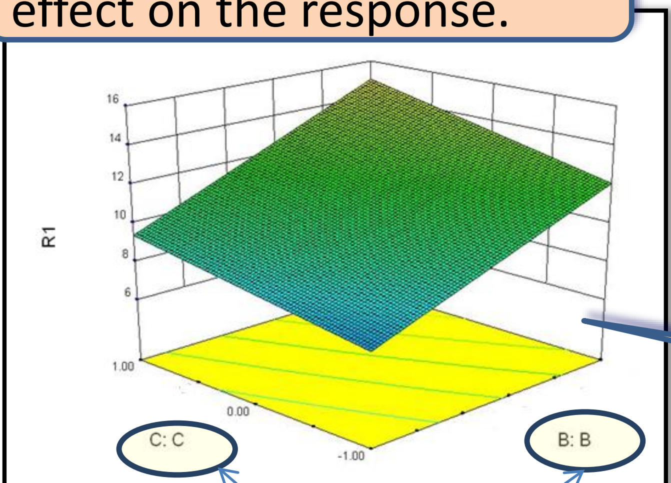


Factor D has no significant effect on the response.

Analysis of Variance Table for Case I (2_V^{5-1}Fractional Factorial); p-value <0.1

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	627.68	6	104.61	139.00	< 0.0001	significant
A	*216.38*	*1*	*216.38*	*287.51*	*< 0.0001*	
B	*303.20*	*1*	*303.20*	*402.85*	*< 0.0001*	
C	*94.92*	*1*	*94.92*	*126.11*	*< 0.0001*	
E	*0.27*	*1*	*0.27*	*0.36*	*0.5513*	
BC	*3.35*	*1*	*3.35*	*4.45*	*0.0393*	
BE	*9.56*	*1*	*9.56*	*12.71*	*0.0007*	
Residual	*42.90*	*57*	*0.75*			
Lack of Fit	*19.01*	*25*	*0.76*	[illegible]	*0.4742*	*not significant*
Pure Error	*23.89*	*32*	*0.75*			
Cor Total	*670.58*	*63*				

F-values consistent with complete randomization.



Continuous

Reduced Empirical Model (Coded Factors)

$$R = I + x_1A + x_2B + x_3C - x_4E + x_{23}BC - x_{25}BE$$

$R^2 = 0.9360$ Adj. $R^2 = 0.9293$ Pred. $R^2 = 0.9193$ Adeq. Precision = 39.2

Continuous factors yield response surfaces.

Experimental Design Approach

Step 6 – Diagnostics (Case II)

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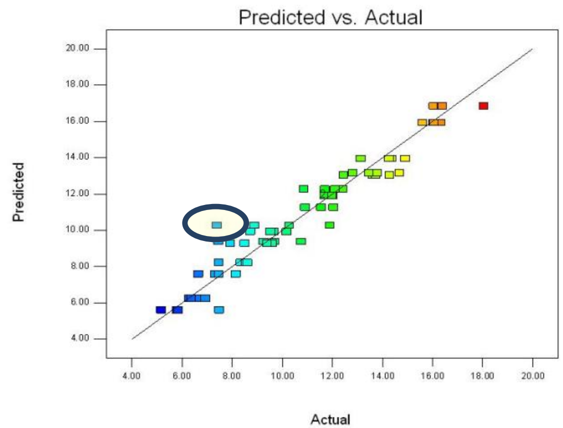
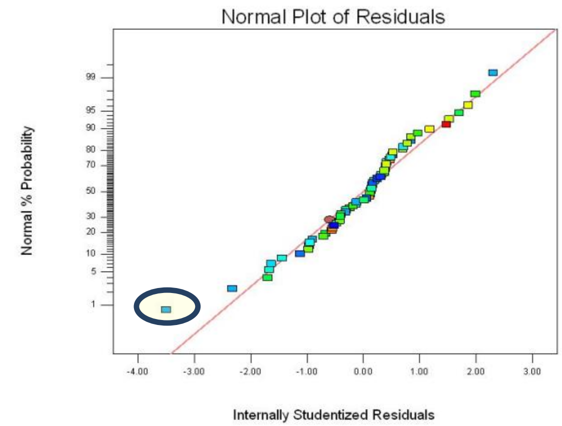
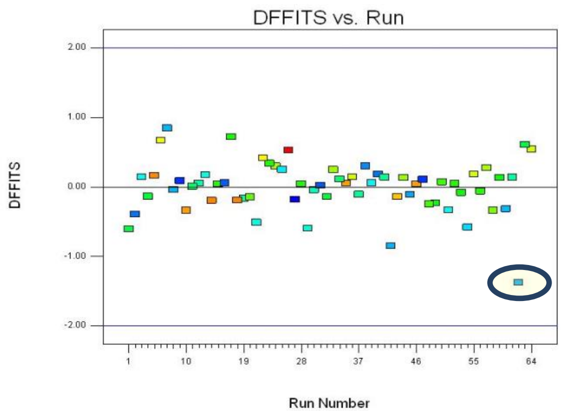
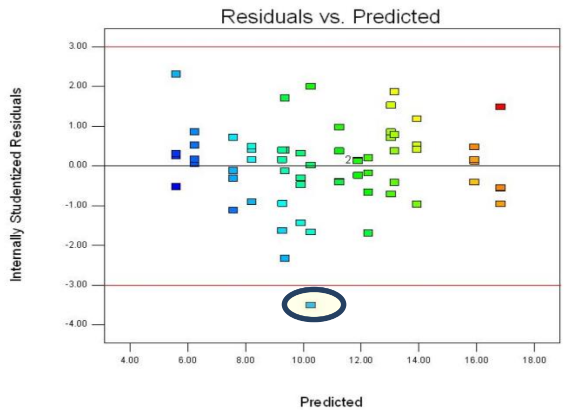
Design

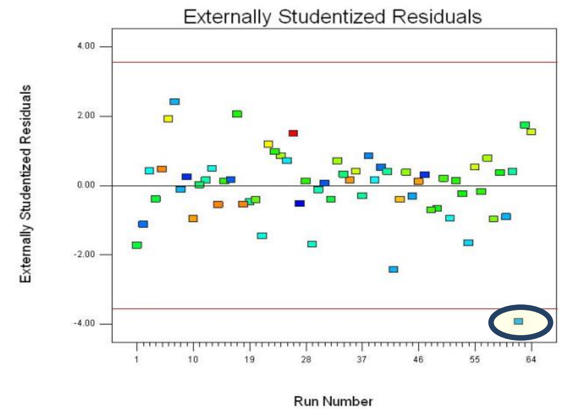
2^5 Factorial
2 Replicates











Validating the data and the statistical assumptions.

Experimental Design Approach

Step 6 - Statistical Analysis (Case III)

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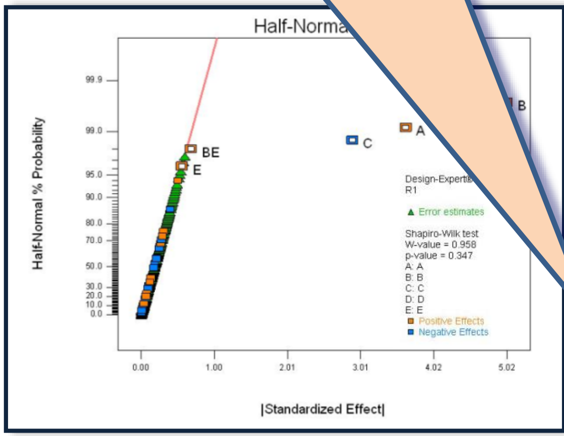
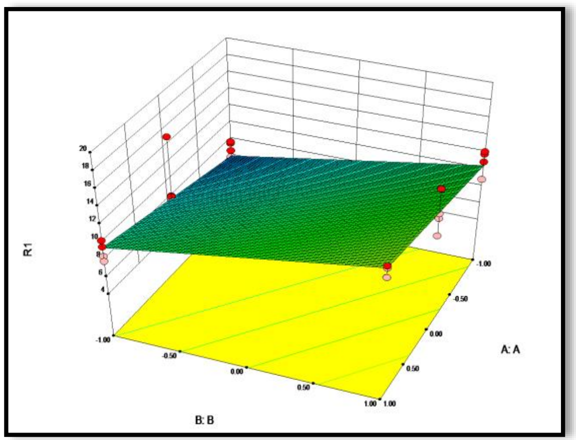
Factors D and E are not significant.





Analysis of Variance Table for Case III (4 x 2^5 Factorial + Center Points); p-value < 0.1

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	1524.59	5	304.92	141.81	< 0.0001	significant
A	*423.11*	*1*	*423.11*	*196.78*	*< 0.0001*	
B	*636.09*	*1*	*636.09*	*295.83*	*< 0.0001*	
C	*191.10*	*1*	*191.10*	*88.88*	*< 0.0001*	
E	*2.90*	*1*	*2.90*	*1.35*	*0.2467*	
BE	*15.04*	*1*	*15.04*	*7.00*	*0.0089*	
Curvature	*22.76*	*8*	*2.84*	*1.32*	*0.2346*	*not significant*
Residual	*382.73*	*178*	*2.15*			
Lack of Fit	*52.47*	*34*	*1.54*	*0.67*	*0.9112*	*not significant*
Pure Error	*330.26*	*144*	*2.29*			
Cor Total	*1930.08*	*191*				

Reduced Empirical Model (Adjusted, Coded Factors)

$$R = I + x_1A + x_2B - x_3C + x5E + x_{25}BE$$

$R^2 = 0.7899$ Adj. $R^2 = 0.7843$ Pred. $R^2 = 0.7775$ Adeq. Precision = 40.9

* Ref: Design Expert 8.0.7.1

Experimental Design Approach

Step 6 - Confirmation

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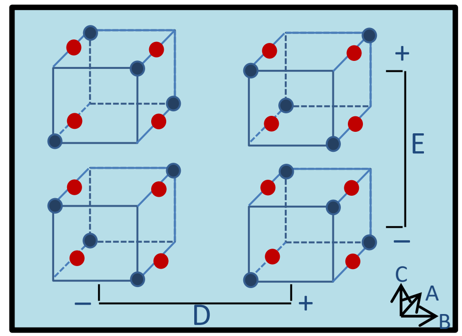
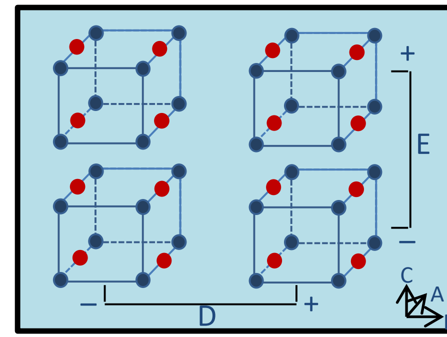
Save a few runs for confirmation.

Empirical Error					
A	**B**	**C**	**E**	**Case I**	**Case II**
0	-1	-1	-1	13.8	5.1
0	-1	-1	1	10.9	0.7
0	-1	1	-1	9.0	3.0
0	-1	1	1	0.2	1.6
0	1	-1	-1	7.1	1.6
0	1	-1	1	4.3	1.4
0	1	1	-1	13.6	0.6
0	1	1	1	3.2	2.0
Average				7.8	2.8

Case I



Case II



- Factor A was a 3-level factor.
- The designs for Case I and Case II used only the high and low settings (in blue), and not the center points (in red).
- The center points were used for confirmation.
- The Empirical Error is the difference between the average (5 runs) at the center points and the respective model predictions for those factor settings.

" All models are wrong, but some are useful. "

George Box

Experimental Design Approach

Test 2

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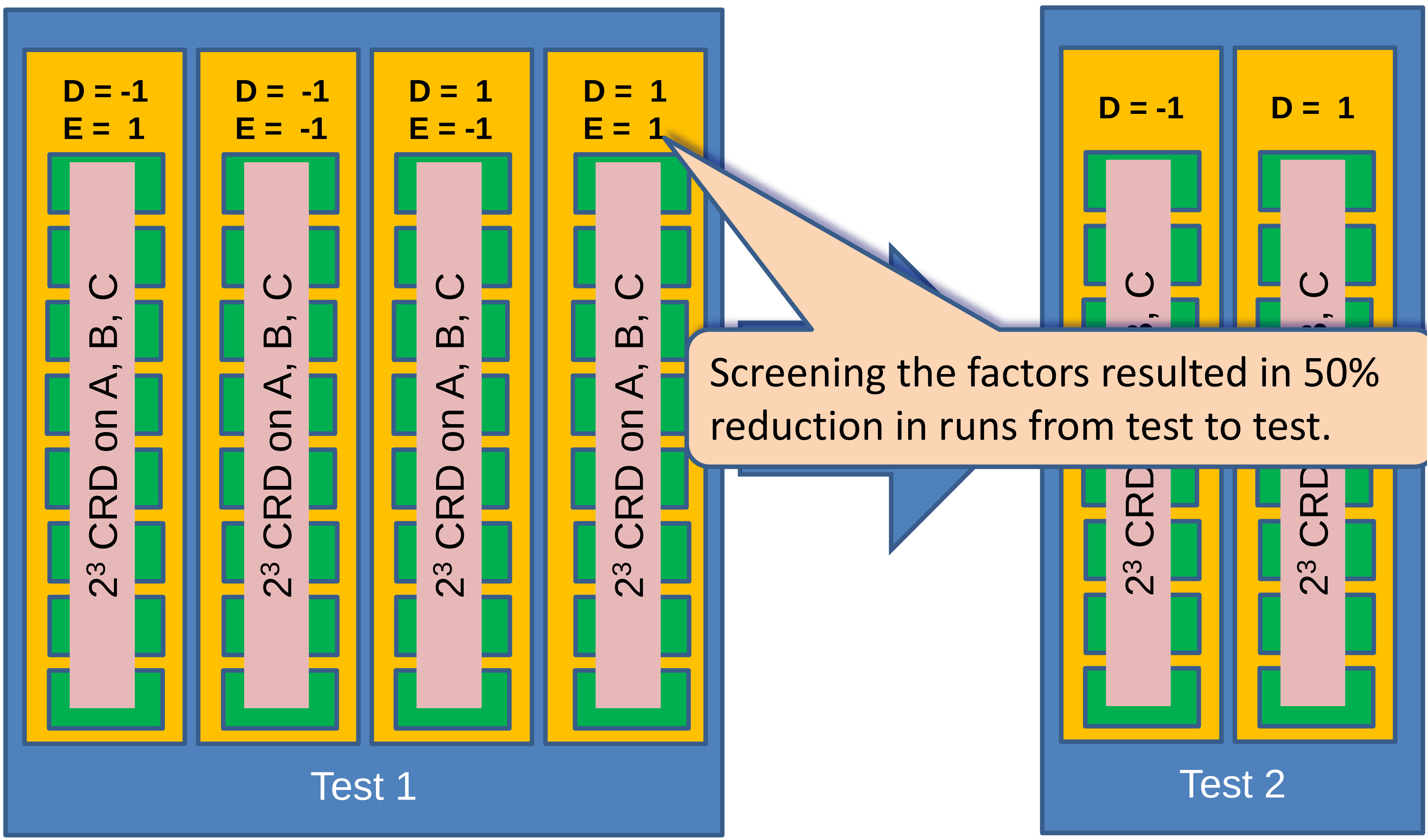

Experimental Design Approach

Step 6 - Statistical Analysis (Test 2, Case II)

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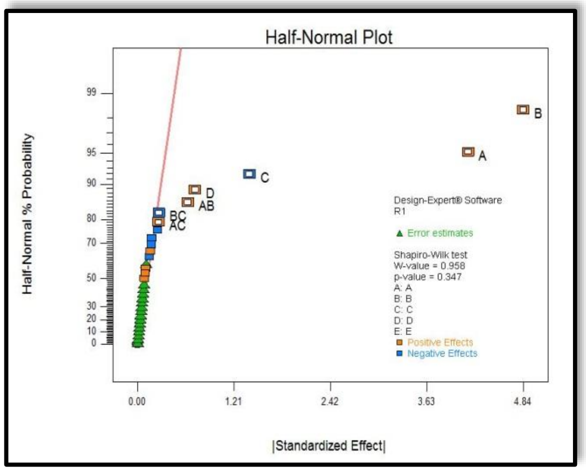

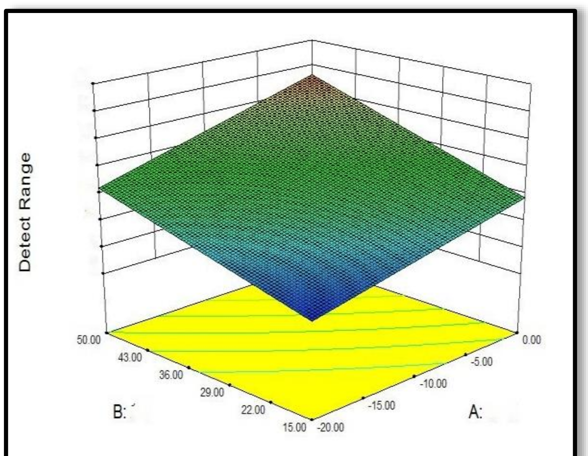

Analysis of Variance Table for Test 2 (2×2^4 Factorial); p-value < 0.1

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	350.07	7	50.01	310.41	< 0.0001	significant
A	*137.99*	*1*	*137.99*	*856.50*	*< 0.0001*	
B	*187.65*	*1*	*187.65*	*1164.74*	*< 0.0001*	
C	*15.83*	*1*				
D	[illegible]					
AB	*3.23*	*1*	*3.23*	*20.06*	*0.0002*	
AC	*0.58*	*1*	*0.58*	*3.57*	*0.0710*	
BC	*0.59*	*1*	*0.59*	*3.67*	*0.0674*	
Residual	*3.87*	*24*	*0.16*			
Lack of fit	*1.68*	*8*	*0.21*	*1.53*	*0.2231*	*not significant*
Pure Error	*2.19*	*16*	*0.14*			
Cor Total	*353.93*	*31*				

Factors D now is significant.

Reduced Empirical Model (Coded Factors)

$$R = I + x_1 A + x_2 B - x_3 C + x_4 D + x_{12} AB + x_{13} AC - x_{23} BC$$

$R^2 = 0.9891$ Adj. $R^2 = 0.9859$ Pred. $R^2 = 0.9806$ Adeq. Precision = 55.5

* Ref: Design Expert 8.0.7.1

Experimental Design Approach
Step 6 - Statistical Analysis (Comparison)

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ANOVA Table for the split-plot experiment (Test 1 vs. Test 2)

Source	dof	SS	MS	F_0	F	Significant
R (Test)	1	16.02	16.02	330.90	10.13	Significant
WP Error	2	0.10	0.05	0.35	*	
A	1	119.74	119.74	872.03	4.38	Significant
B	1	147.15	147.15	1071.65	4.38	Significant
C	1	27.08	27.08	197.25	4.38	Significant
AB	1	3.45	3.45	25.09	4.38	Significant
AC	1	0.00	0.00	0.03	4.38	Not significant
BC	1	1.16	1.16	8.41	4.38	Significant
RA	1	0.26	0.26	1.89	4.38	Not significant
RB	1	1.73	1.73	12.60	4.38	Significant
RC	1	2.99	2.99	21.77	4.38	Significant
SP Error	19	2.61	0.14	*	*	*
	31	322.28				

A split-plot design was used to compare detection performance between the tests. Factor R is significant—there is a difference between the radar systems.

Summary

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- Experimental design is the integration of well defined and structured scientific strategies for gathering empirical knowledge using statistical methods for planning, designing, executing, and analyzing a test.
- Experimental design provides a comprehensive understanding of the trade-offs in the techno-programmatic domains: risks, cost, and utility of information.
- Experimental design can help reducing test assets, shortening the test schedule, and providing more information to the warfighter and decision makers.
- Experimental design adds rigor and discipline to T&E.

Conclusions

What could we have done differently?

Only 16 runs

+ center points

+ axial points

(maybe)

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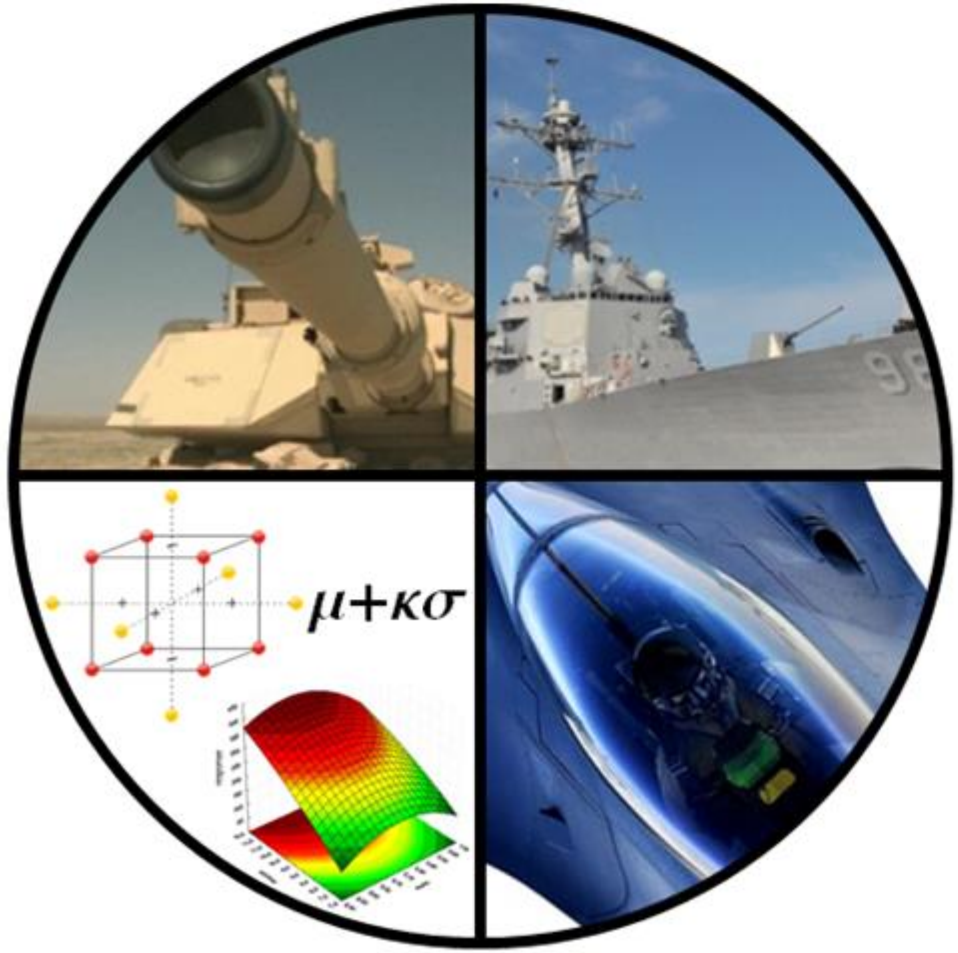

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Knowledge, insight, and value