Impact of Test Time on Estimation of Mean Time Between Failure (MTBF)

Authored by: Lenny Truett, Ph.D.

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The goal of the STAT COE is to assist in developing rigorous, defensible test strategies to more effectively quantify and characterize system performance and provide information that reduces risk. This and other COE products are available at www.AFIT.edu/STAT.

STAT Center of Excellence
2950 Hobson Way – Wright-Patterson AFB, OH 45433

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Revision 1, 3 Oct 2018: Formatting and minor typographical/grammatical edits.

Introduction

Testing is often required to evaluate the mean time between failure (MTBF) for a component or system. If we could test for an infinite amount of time, we could determine the true MTBF. Instead, we have to set a specific test time and compute an observed MTBF. Since an observed average is only an estimate of the true mean, it is important to report the MTBF as an interval of plausible values. This is often accomplished by calculating a confidence interval or a lower confidence bound of the system MTBF. This best practice gives you a simple way to quickly understand the effect of total test time on confidence intervals and the lower bounds for a process with a constant failure rate. It will also highlight that this approach may not be feasible to demonstrate a required MTBF with a specified confidence levels for many practical situations. This guide will help you determine if this approach is feasible or if you should explore other methodologies to evaluate the MTBF.

Formulation

This best practice examines the case where the failure rate is constant and therefore the distribution of the system failure time is exponential. The Chi-Square (χ^2) distribution can be used to calculate the confidence bounds for the system MTBF. The formulas for the one-sided confidence interval (MTBF lower limit) and two-sided confidence interval vary slightly for a time truncated test (based on a predetermined test time) versus a failure truncated test (based on a predetermined number of failures). In Equations 1 – 4, T is the total test time, α is the acceptable risk of type I error (1- confidence), and n is the number of observed failures. This best practice will focus on the time truncated methods since this is what most Department of Defense (DoD) tests implement.

$$\left[\frac{2T}{\chi^2_{\alpha,2n}}\right] \qquad (1)$$

One-Sided Failure Truncated

$$\left[\frac{2T}{\chi^2_{\alpha,2n+2}}\right] \qquad (2)$$

One-Sided Time Truncated

$$\left[\frac{2T}{\chi^2_{\alpha/2,2n}}, \frac{2T}{\chi^2_{1-\alpha/2,2n}}\right] \qquad (3)$$

Two-Sided Failure Truncated

$$\left[\frac{2T}{\chi^2_{\alpha/2,2n+2}}, \frac{2T}{\chi^2_{1-\alpha/2,2n}}\right] \quad (4)$$

Two-Sided Time Truncated

A calculator for all four equations can be found at
http://reliabilityanalyticstoolkit.appspot.com/confidence_limits_exponential_distribution

Estimating a Confidence Interval for MTBF

Figure 1 is a normalized graph that can be used to quickly estimate the confidence interval for any MTBF for a time truncated test when the calculator is not available. The vertical axis shows the normalized MTBF and the horizontal axis is the number of observed failures.

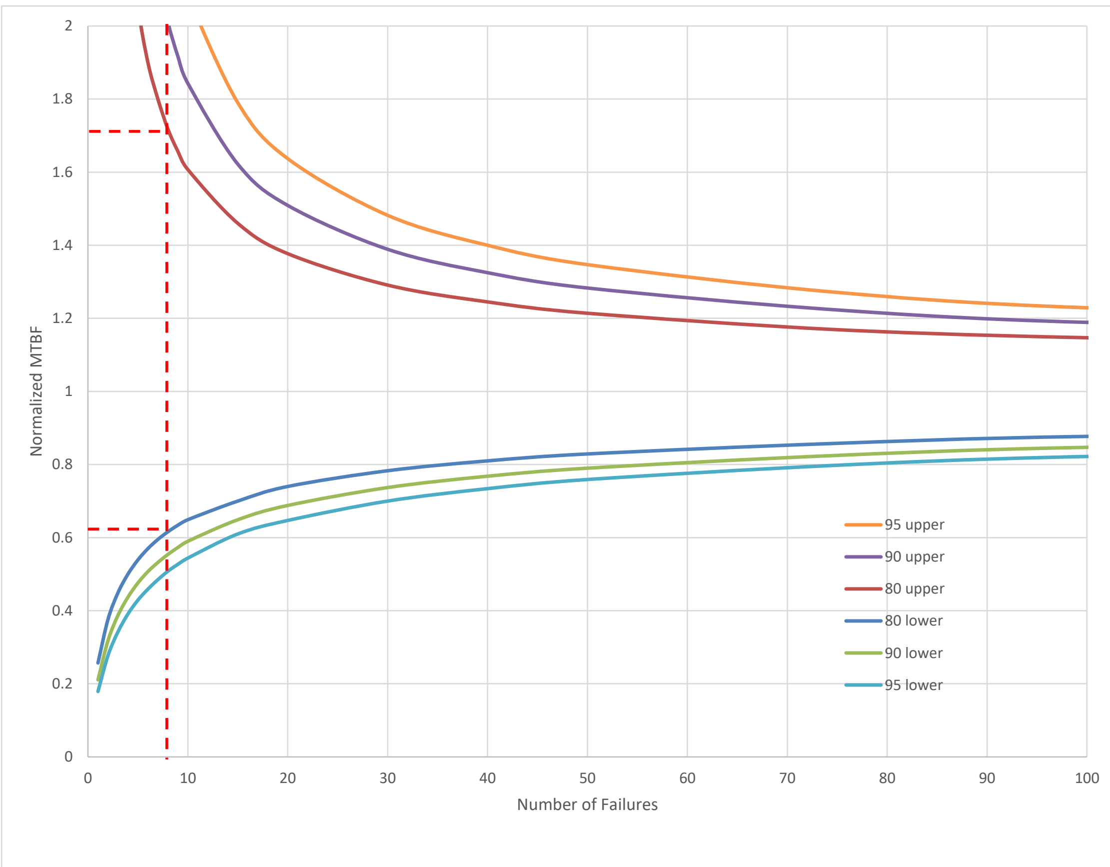

Figure 1: Normalized MTBF versus Number of Failures for 2-sided Confidence Interval

For any given MTBF and observed number of failures, you can quickly estimate the confidence interval by following the horizontal axis until you get to the observed numbers of failures. Then go vertically and determine the values where this line crosses the curves for the desired value of α. Read the lower and upper values from the vertical axis and multiply these numbers by your observed MTBF. For example, if you test for 300 hours and observe 8 failures, your observed MTBF would be 300/8 or 37.5 hours. On the chart (shown in red dashed line), for n = 8, the 80% confidence interval is approximately 0.62 and 1.72. So the 80% confidence interval would go from (37.5 * 0.62) to (37.5 * 1.72), or 23.25 to 64.50. This is a wide estimate for the true mean, and has a type I error rate of 20%. The only way to make the bounds on the interval estimate of the true MTBF smaller is to test longer.

Estimating a Lower Bound for MTBF

In the DoD, testing is usually focused on determining the lower bound of the MTBF. This lower bound is usually required to be at or above the requirement with some confidence level. The DoD commonly uses an α of 0.2, even though other industries would consider this to be quite high. The lower bounds for various values of α for a time truncated test are shown in Figure 2.

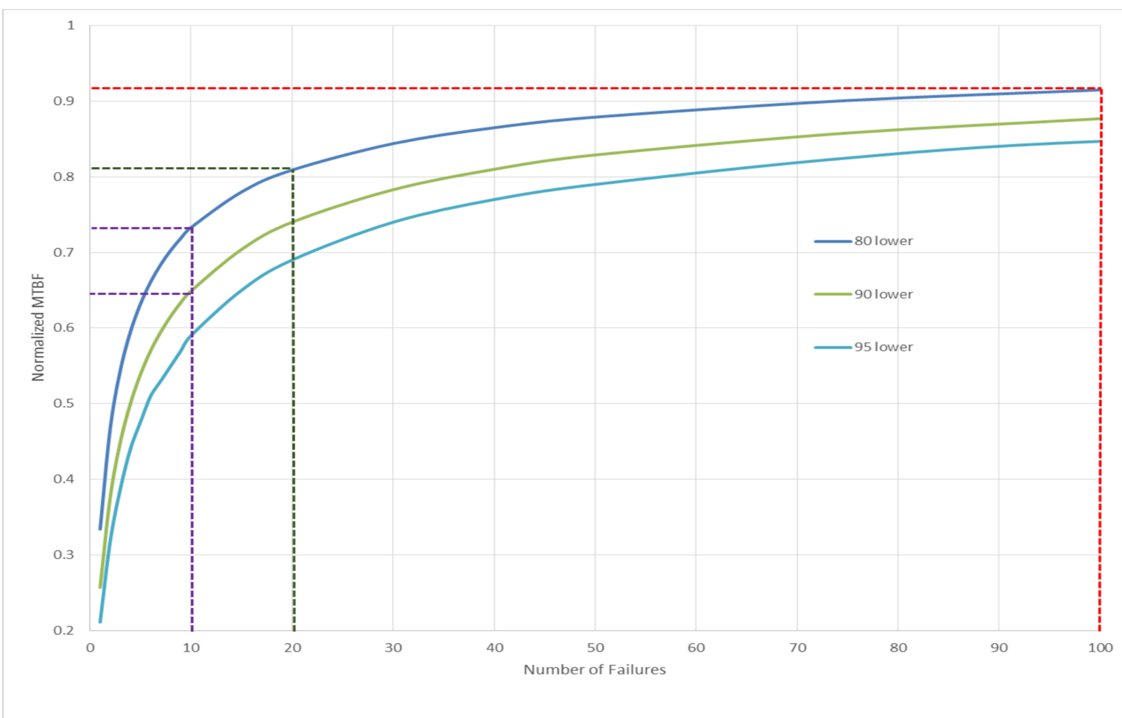

Figure 2: Normalized MTBF versus Number of Failures for Lower Bound

Estimating the Required Test Time

Suppose you need to demonstrate that a system meets a specific MTBF requirement with a specified level of confidence. You know that the observed MTBF will have to be higher than the required MTBF in order for the lower confidence bound to be at least as large as the requirement. The difference between the observed MTBF and the required MTBF will reduce as the number of failures increase. If you estimate how much margin you hope to observe, you can estimate the required number of observed failures. You can also estimate how much margin you need for a given number of observed failures. For example, assume you have an MTBF requirement of 64 hours and based on information from previous testing as well as insight from the manufacturer, you believe that you can reasonably demonstrate an MTBF of 70 hours. 64 / 70 equals 0.914. Therefore, if you draw a horizontal line from 0.914 (shown in Figure 2 as the red dashed lines), it will cross the various curves at the number of failures you need to observe for the lower confidence bound to meet the requirement. For $\alpha = 0.2$, you would need 100 failures (or less) and a total test time of 7000 hours, to observe a MTBF that has the 80% confidence lower bound equal to 64 hours. Another way to use this chart (shown in Figure 2 as green dashed lines) would be to examine a test period long enough to observe 20 failures. The lower bound for 80% confidence is 0.809. Divide your requirement of 64 by 0.809 to get 79.11. Therefore for you would need 20*79.11 or 1582.2 hours test time with 20 maximum observed failures and observed MTBF of 79.11 hours to demonstrate a MTBF of 64 hours with 80% confidence. As you can see from these examples, you need to observe an MTBF significantly higher than the requirement for this type of evaluation. For lower numbers of failures, this situation is even more demanding (shown in purple dashed lines). If you had an observed MTBF of 70, but only saw 10 failures in 700 hours of testing, the 80% confidence lower bound would be 0.733 * 70 or an MTBF of 51.31 hours. The 90% lower bound would be 0.649 *70 or an MTBF of 45.43 hours. Clearly, this method of estimating the lower bound is not appropriate when the observed average is expected to be near the minimum requirement and it is only applicable for a constant failure rate.

Alternative Approaches

When the number of failures that will be observed in the available test time is limited, or the margin between the required and observed MTBF is expected to be small, other test approaches should be considered. A sequential reliability test establishes an acceptance limit and a rejection limit and testing continues until one is reached. This may require additional test time, but it does not force a decision to be made at a predetermine time. Bayesian methods have been increasing in popularity because they can incorporate previous testing or previous knowledge about the reliability of a system under test. While the theory is simple, the application can be tricky and should only be attempted when expert assistance is available. The STAT COE has developed a best practice to introduce Bayesian methods that can be found at
https://www.afit.edu/stat/statcoe_files/Practical_Bayesian_Analysis_for_Failure_Time_Data_Best_Practice.pdf

Summary and Conclusion

For the assumption of a constant failure rate, you can quickly estimate the bounds for confidence intervals and lower confidence bounds for a given test time and number of failures. From these figures, you can examine the effect of increasing test time or changing the anticipated MTBFs. If you believe that you may not be able demonstrate the MTBF necessary to have the lower bound above the requirement because you do not have the margin or test time required, you may want to consider other approaches to obtain an estimate of the MTBF for your system.